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DECOUPLED FEEDFORWARD-FEEDBACK CONTROL OF A BINARY DISTILLATION COLUMN

BY

© DOUGLAS W. LIESCH

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT TO THE REQUIREMENTS FOR THE DEGREE
OF MASTER OF SCIENCE IN CHEMICAL ENGINEERING

DEPARTMENT OF CHEMICAL ENGINEERING
EDMONTON, ALBERTA
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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled DECOUPLED FEEDFORWARD-FEEDBACK CONTROL OF A BINARY DISTILLATION COLUMN submitted by Douglas W. Liesch, B.Sc. (Ch.E.), in partial fulfilment of the requirements for the degree of Master of Science.

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Supervisor

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Date ..August 7, 1974..

ABSTRACT

An approach to solving the problems encountered in dual quality control of distillation columns is tested experimentally using a pilot scale distillation column with a methanol-water system. The problems dealt with include interaction between the overhead and bottoms composition control loops, measurement noise and sluggish response of the controlled variables.

To lessen these problems first the physical configuration was studied. An experimental program was carried out to evaluate which control variable for overhead composition control, reflux or distillate flow, gave better performance. It was found that depending on the type of disturbance either could be better for the case where bottoms composition control was not operative. The distillate flow was chosen as the control variable for the subsequent control studies where both terminal compositions were regulated.

A control technique that requires relatively simple modelling to obtain decoupling to reduce interaction and feedforward control to speed disturbance attenuation was chosen for this study. A linear model is required for use in the formulation of the control law.

Distillation modelling was examined with respect to mechanistic type models using energy and mass balances and physical data. A concentration model was chosen for the control calculations as it was found to agree with experimental results from the equipment better than an enthalpy model that was tried.

The final control law is calculated using a model reduced from the original tenth order concentration model to fourth order by modal reduction. This was discretized to enable implementation with a digital computer.

The performance of the system was simulated and the results predict good disturbance attenuation.

Experimental results are presented for the control tests with the equipment including those in which integral feedback was added. The decoupling in the control function facilitates on-line tuning of feedback functions. Test results presented indicate that the control scheme is successful for regulatory control.

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CHAPTER 1

INTRODUCTION

Control of a binary distillation column entails several problems that make simple single variable feedback control unsatisfactory. First, the terminal compositions respond slowly to disturbances in the feed which means that a large upset is present in the internal variables before control action can be taken. This gives slow recovery from a disturbance. Second, the system exhibits highly non-linear behavior so that control gains must be changed to allow for the change in process gain when the system is away from the study state due to an upset. This also means that a linear model is not adequate to predict the response of the system for a large change in input conditions. Another difficulty with single variable feedback control stems from noise in the system or unreliable measurements which may result in incorrect control action. The final unfavorable phenomenon is the interaction that takes place between the composition control loops on the overhead and bottoms streams. This reduces the stability of the control loops, degrades the performance and makes it extremely difficult to tune the controllers as tuning of one affects the other.

The object of this work is to develop a practical control scheme for a binary distillation column for regulation of the terminal compositions and to present experimental results using the scheme on the pilot scale distillation column at the University of Alberta. The development is carried out in three stages, namely: (1) an evaluation of the choice of manipulated variable for control of the overhead composition, (2) modelling of the distillation column, and (3) the calculation and implementation of the control law.

The first stage of this work is an experimental study in which the reflux flow and the distillate flow were tested as the manipulated variable in order to determine which gives better control of overhead composition. Past control studies at the University of Alberta on the column have used the reflux flow exclusively, and, as this method has come into question (5,19,27,33) it is compared experimentally to the alternative distillate flow for the case where only the overhead composition is controlled while the bottoms composition controller is left on manual.

The second phase of this study is the derivation of a model suitable for use in establishing a control law. The control method developed in this study is restricted to a linear model. A model in linear state space format which meets that criterion was developed by McGinnis (28). He used a twentieth order model with the states being the liquid enthalpies and flowrates based on the dynamic energy balance and assuming a first order lag for the liquid flowrates from tray to tray. He assumed that the vapor flow remained constant and that the condenser load was a free input. This model gave reasonable results when the experimental change in the condenser load was added to the disturbance input to the simulation. It should be noted that the condenser load is used to control the column pressure which is assumed constant so the condenser load is determined by the other inputs. For this reason other methods of representing the vapor flow and condenser load were considered. Non-linear models based on the unsteady state material and energy balances are examined along with the assumptions necessary to make them solvable. The simulation results from the models are then compared to experimental results. The model that best predicted experimental results was linearized around steady state conditions in order to enable its use in the control of the column.

The final portion of the study is devoted to obtaining a control scheme that can be implemented on the column and to testing it experimentally. The control law was based on the method proposed by Greenfield and Ward (13,14). This method gives feedforward control with input and output decoupling. The decoupling removes the interaction and allows the addition of simple feedback control loops. These loops can be easily tuned on-line so that the errors from modelling and linearization can be compensated for. Berry (2) developed a decoupled feedback control scheme that gave excellent control. He started with an experimental first order plus time delay transfer function model but found that tuning was required for both feedback and decoupling elements. McGinnis' multivariable scheme, which was based on a second order discrete model found by reducing the twentieth order model (by a least squares fit approach), was also unsuccessful until he did some tuning. In this study the formulation is attempted directly using the tenth order linear state space model with the concentrations as the states. Since the control is carried out using a digital computer the discrete version of the control law is also established. In addition some model reduction is used to simplify the control action and to remove unnecessary measurements.

The emphasis of this study is the development of a practical method of control for a distillation column. The questions to be answered include the desirability of expending considerable effort in developing an accurate model as opposed to an approximate model that gives some description of all the natural phenomena and can be obtained relatively easily, the importance of proper control scheme configuration and the advantage of making specific allowance for on-line tuning.

CHAPTER 2

CONTROL VARIABLE FOR DISTILLATE COMPOSITION

2.1 Introduction

The initial step in designing a control system for a process consists of choosing the variables that can be manipulated to achieve the desired control. The system used in this study, a pilot scale binary distillation column as described by Pacey (35) and Svrcek (48) has five primary variables which are to be controlled. These are: pressure (in the condenser), liquid levels in the condenser and reboiler and the two terminal compositions. The feed conditions are considered to be the source of disturbance. The control configuration that has been used on this system by previous workers (2,28,35,48) utilizes condenser load to control the pressure, bottoms flow to control the reboiler level and distillate flow to control the condenser level. This leaves the reboiler load and the reflux flow to control the terminal compositions. This type of control system is termed energy balance composition control because the split between overhead and bottoms can be changed only indirectly through a change in reboiler load or reflux rate as the exit streams are both on level control.

Recently energy balance composition control has been questioned. Shinskey (46) states that since both product streams are on level control there is no direct way of manipulating the material balance and so better product control could be obtained by manipulating a product stream. Other studies (5,19,27,33) have included tests of the two different systems on various distillation columns and shown that the control schemes using a product flow to control composition, termed direct material balance control of composition, gave improved control behavior for the systems tested.

The arguments presented in favor of direct material balance control are inherent reflux control, less interaction and greater stability for the controlled system. Inherent reflux control as explained by Nisenfeld (33) involves the reaction of the system to a heat loss disturbance due to something like a sudden decrease in external temperature. The system using the distillate flow to regulate the overhead composition would cause a decrease in the reflux rate (external reflux) but the total reflux, internal and external, would remain about the same so that the overhead composition should not be disturbed. However, in the case of the energy balance control system, the reflux could not change until the overhead composition changed and thus the controlled variable would be disturbed.

The claim of a decrease in interaction between the overhead and bottoms composition control has been supported by analysis using the relative gain matrix proposed by Bristol (5). The elements in this matrix are the ratio of the open loop change in the controlled variable to the change when the other controlled variables are held constant under control for a change in the specified manipulated input. This was used to measure the steady state interaction between the bottoms and overhead composition control using alternately the reflux then the distillate to control the overhead composition. When the reboiler load is assumed to be characterized by the separation factor (Shinskey (46)) the relative gain for the two alternate overhead composition control loops can be related as given by Juantoreno and Romeo (19).

$$\lambda_{x_D,R} = \frac{\lambda_{x_D,D}}{1 - 2\lambda_{x_D,D}} \qquad 2.1$$

where $\lambda_{x_D,R}$ = the relative gain between overhead composition and reflux flow

$\lambda_{x_D,D}$ = the relative gain between overhead composition and distillate flow

Nisenfeld and Shultz (34) developed an interactive index which can be stated as,

$$I_j = \frac{1 - \lambda_{i,j}}{\lambda_{i,j}} \qquad 2.2$$

where I_j = the interactive index for j

$\lambda_{i,j}$ = the relative gain between i and j

Shinskey (46) implies that interaction for the energy balance system is always worse (except for $\lambda_{x_D,D} = 0$ or 1 which are physically unrealizable) but by substituting equation 2.1 into 2.2 it can be seen that the interactive index is the same for both systems. He also states that if $\lambda_{x_D,R}$ is less than zero, control is not possible by the energy balance system. This is true but even for direct material balance control the overhead variable is best controlled by the reboiler load. Thus it can be seen that the steady state analysis involving the relative gain matrix is not as useful as first represented in choosing the control configuration for overhead composition.

One transient effect which causes interaction and degrades control is the inverse response undergone by the overhead composition for an increase in reboiler load which involves an initial response of the control variable in the opposite direction to the final steady state deviation. Rosenbrock's (41) explanation of this phenomenon states that the plate overflow is suddenly increased due to an increase in the vapor flow which functions as an apparent increase in reflux which initially causes the composition in the column to deteriorate. The energy balance control scheme initially causes an increase in the distillate flow while the external reflux remains constant. The direct material balance system immediately increases the external reflux so the initial decrease in concentration of the overhead should never occur thus eliminating the inverse response.

The third feature of the control system using the distillate to control overhead composition is the increased stability because of direct control of the smaller flow. The reflux ratio of high purity product industrial columns is often over ten. Shinskey (47) states that in this case the distillate flow is too small to control the level in the condenser accumulator. Harriot (16) stated that a small error in the reflux flow would cause a much larger relative error in the distillate rate and thus a large disturbance to the material balance for the system with distillate flow for level control however switching the reflux flow for level control would minimize this problem.

In view of the arguments presented in favor of direct material balance composition control it is interesting to consider why the energy balance system has had wise usage. This can be explained by the fact that the reflux is the only variable that can be used to directly alter the vapor to reflux balance in the upper end of a distillation column which in turn directly changes the overhead composition. The alternate system must rely on the condenser accumulator level controller to change the reflux flow after the distillate flow has been changed. It is clear that at least for setpoint control the overhead composition should respond faster when the reflux is directly changed by the composition control loop.

It is not possible to specify the best control configuration for all columns because each different distillation column has its own particular characteristics. The pilot scale distillation column and the operating conditions are such that they would diminish the stated advantages of the direct material balance system. In particular the reflux ratio is less than two so that the sensitivity of the reflux and distillate flows do not differ greatly. The fact that the column is located in the laboratory means that no great change in heat loss can be anticipated. Also, the holdup of the condenser accumulator is about the same as that on the trays so that the dynamic lag is small hence both reflux and distillate flows respond at almost the same time.

In view of the uncertainty of the virtues of the alternate systems the two systems were tested. The experiments were carried out with overhead composition controlled directly by either reflux or distillate flow with the other flow maintaining level control of the condenser accumulator level. Reboiler load was maintained constant for these tests.

2.2 Experimental Program

The pilot plant distillation column used in this study has sufficient instrumentation so that either of the two alternate control schemes for overhead composition control as shown in Figures 2.1 and 2.2 can be implemented simply by connecting appropriate pneumatic lines.

The first tests were carried out using the Foxboro capacitance Dynalog which has a pneumatic-proportional plus integral controller coupled to the capacitance (composition) measurement. A second set of tests were carried out using the DDC (Direct Digital Control) package on the IBM 1800 digital computer to replace the controller function of the

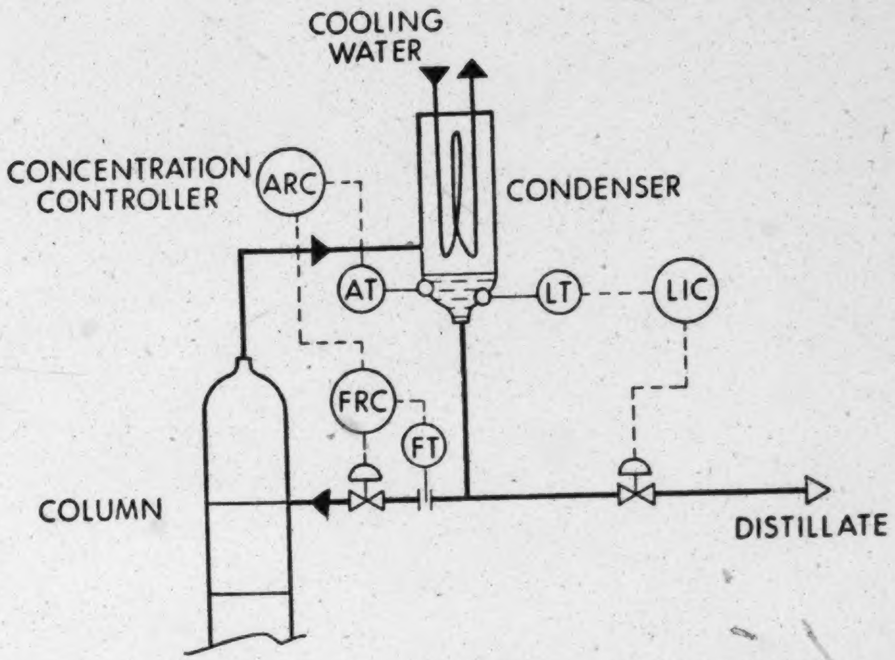


FIGURE 2.1: Schematic of Overhead Composition Control Configuration Using Reflux Flow (Energy Balance Control)

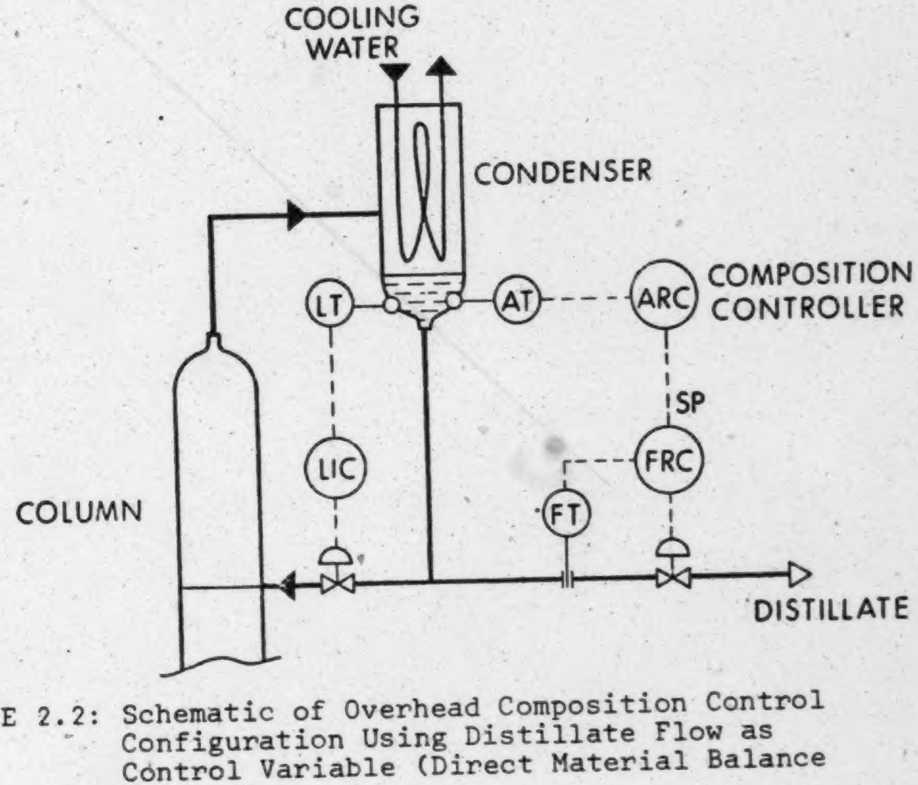


FIGURE 2.2: Schematic of Overhead Composition Control Configuration Using Distillate Flow as Control Variable (Direct Material Balance Control)

Dynalog. This was done to test the effect of discrete control and to obtain a more easily reproducible set of results. The reason that DDC control is easier to reproduce stems from the fact that input to the control function is in weight percent and output is in pounds per minute of flow rather than instrument air pressures. Using the physical variables gives absolute measurements while the analog signals (pressures) are relative so that a calibration change requires changing the controller constants to maintain comparable results.

For these tests the distillation column was subjected to feed flow changes and to a decrease in feed temperature. The step changes in feed flow were made to enable comparison of performance to that obtained in previous control experiments (2,28,35,48). The feed temperature disturbance was used to show the effect of an enthalpy change at the middle of the column in order to simulate a change in heat loss.

Two open loop responses to steam flow changes (changes in reboiler load) were also performed. These are included to indicate the degree of interaction present between reboiler load (used for bottom composition control in a latter phase of this work) and the overhead composition control schemes.

2.3 Experimental Results

The initial tests performed to compare reflux and distillate flow as the manipulative variable with the

Dynalog controller are listed in Table 2.1. Steady state values of the overhead composition, the time integral of the absolute error (IAE) of overhead composition along with the change in the bottoms composition are also shown. The overhead composition responses are plotted in Figures 2.3 to 2.5. Also included for comparison are two runs performed by Pacey. The IAE values indicate the distillate case (distillate flow used to control overhead composition) is generally better. Figure 2.3 shows the effect of a decrease in feed flow from 2.46 to 1.94 pounds per minute. It can be seen for the case where reflux flow is controlling overhead composition that the run performed by Pacey gave poorer control performance even though the two runs were carried out with the same configuration. In addition the initial response of Pacey's run was in the opposite direction from the other two runs which would indicate that the test started nearer to the flooding region. Presumably this behavior resulted because the operating parameters were not exactly the same as those used by Pacey.

The positive feed flow step response depicted in Figure 2.4 shows only small deviation in the overhead composition. It is again in the opposite direction from that of the run performed by Pacey.

The final runs in which the analog controller was used were responses using the alternate overhead composition control configurations to a negative step in feed temperature from 160°F to 145°F. The response for the distillate case

TABLE 2.1

EXPERIMENTAL RESULTS FROM OVERHEAD COMPOSITION CONTROL TESTS USING FOXBORO DYNALOG ANALOG CONTROLLER

RUN	N-1	FB92A	N-6	FB90A	N-9	N-10	N-3
Disturbance A - F = 2.46-1.94 lb/min B - F = 2.46-2.92 lb/min C - T_F = 160-145°F	A	A	A	B	B	C	C
Steady State Distillate Composition (Wt. Pct. MEOH)	96.0	96.0	96.1	96.1	96.5	95.5	95.4
IAE	10.2	17.0	7.9	6.7	3.5	5.1	4.7
Change in Bottoms Composition (Wt. Pct. MEOH)	*	*	*	*	8.2	*	.1
Control By R - Reflux Flow D - Distillate Flow	R	R	D	R	D	R	D
Figure	2.3	2.3	2.3	2.4	2.4	2.5	2.5

* Composition went beyond instrument span.

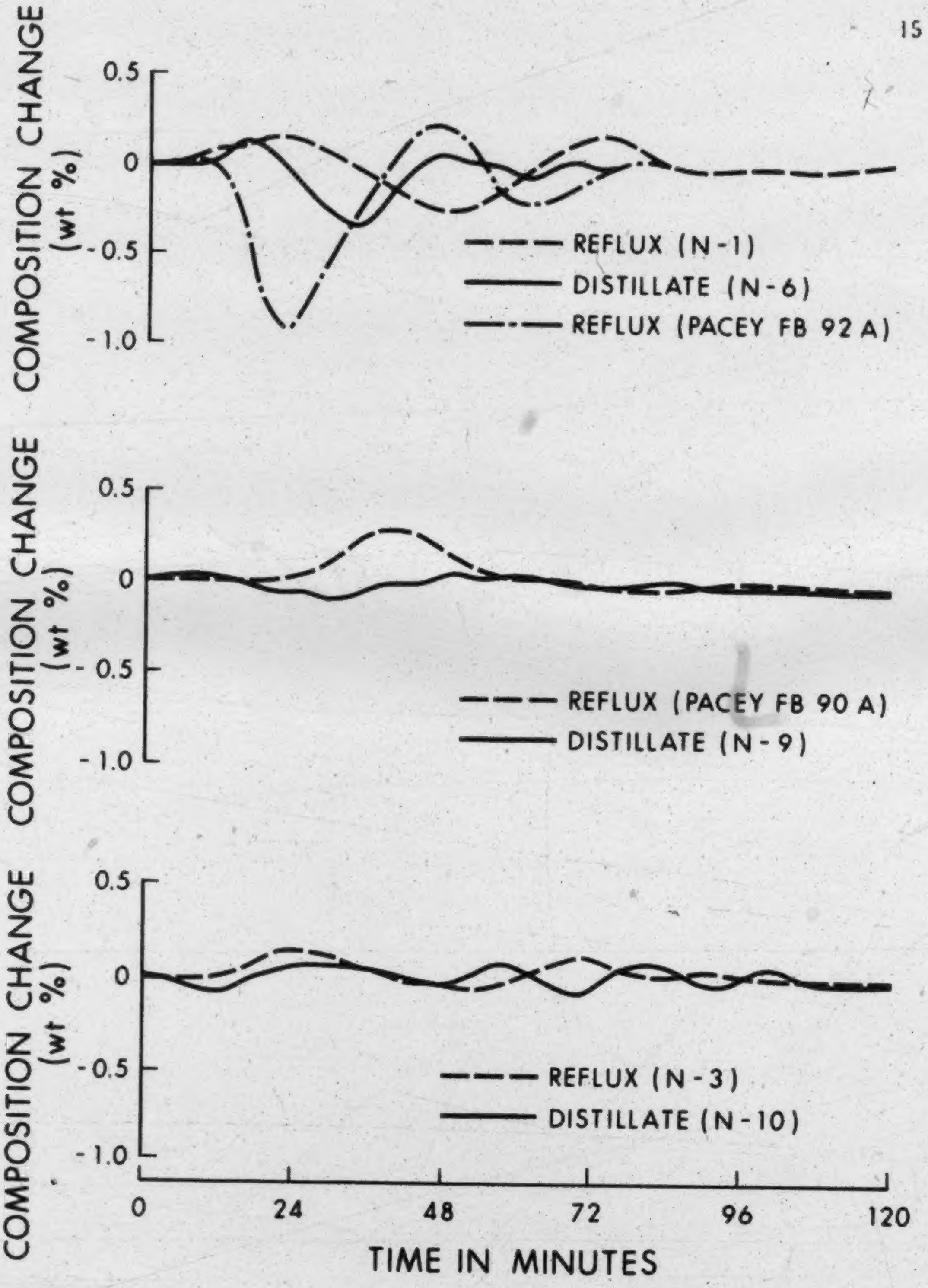


FIGURE 2.5: Experimental Response of Overhead Composition to a Step Decrease in Feed Temperature (160-145°F) Using an Analog Controller

shows some low amplitude cycling. As the control valves do not have positioners this was thought to be due to hysterisis in the reflux flow valve on level control.

The tests carried out using DDC to perform the control function are listed in Table 2.2. This table also includes the steady state and IAE of the overhead composition along with the deviation of the bottoms composition. Table 2.3 gives results from the run with the composition controller on manual.

The response of the overhead composition for the first run using DDC is shown in Figure 2.6. The disturbance used was a feed flow step from 2.4 to 2.64 pounds per minute. On the basis of maximum deviation in the open loop case and the IAE in the closed loop case it follows that reflux is far superior to distillate flow for controlling overhead composition. Comparison of the controlled response with that obtained using analog control shows that the response is different in character which can again be attributed to a change in column operating conditions.

The open loop response of overhead composition for a negative ten percent feed flow change shown in Figure 2.7 exhibits a much larger deviation than occurred for the step change in the positive direction. This behavior clearly shows the nonlinearity of the system. The fact that the closed loop responses for the negative step are unstable although the same controller constants were used as for the positive feed flow change is attributable to this non-

TABLE 2.2

EXPERIMENTAL VALUES FROM OVERHEAD COMPOSITION CONTROL TESTS USING DDC

RUN	A-3	B-4	A-7	B-8	A-11	B-12	B-13	FB80D	B-14	B-15
Disturbance A - F = 2.4-2.64 lb/min B - F = 2.4-2.16 lb/min C - T_F = 161-90°F D - F = 2.46-2.92 lb/min	A	A	B	B	C	C	A	D	D	D
Steady State Distillate Composition (Wt. Pct. MEOH)	97.0	96.0	97.0	97.0	97.0	96.9	96.0	96.1	96.0	96.0
IAE	4.0	16.6	8.5	33.2	14.8	13.0	5.4	8.2	27.9	17.6
Change in Bottoms Composition (Wt. Pct. MEOH)	3.9	3.0	*	*	4.9	3.5	3.7	*	4.4	4.7
Control Variable R - Reflux Flow D - Distillate Flow	R	D	R	D	R	D	D	R	D	D
Figure	2.6	2.6	2.7	2.7	2.8	2.8	-	2.9	2.9	2.9

* Measurement went beyond instrument span.

TABLE 2.3

OPEN LOOP TEST RESULTS FOR OVERHEAD COMPOSITION CONTROL

RUN		A-1	B-2	A-5	B-6	A-9	B-10	A-16	B-17
Disturbance A - F = 2.4-2.64 lb/min B - F = 2.4-2.16 lb/min C - T_F = 161-90°F D - S = 1.85-1.95 lb/min E - S = 1.8-2.035 lb/min		A	A	B	B	C	C	D	E
Steady State Distillate Composition (Wt. Pct. MEOH)	Initial	96.85	97.13	96.9	97.9	96.8	97.3	98.1	96.5
	Final	96.84	97.8	96.0	*	97.8	96.8	97.7	96.4
Change in Bottoms Composition (Wt. Pct. MEOH)		4.9	6.7	*	*	9.3	-1.0	-4.6	-3.3
Control Variable R - Reflux Flow D - Distillate Flow		R	D	R	D	R	D	R	D
Figure		2.6	2.6	2.7	2.7	2.8	2.8	-	-

* Measurement went beyond instrument span.

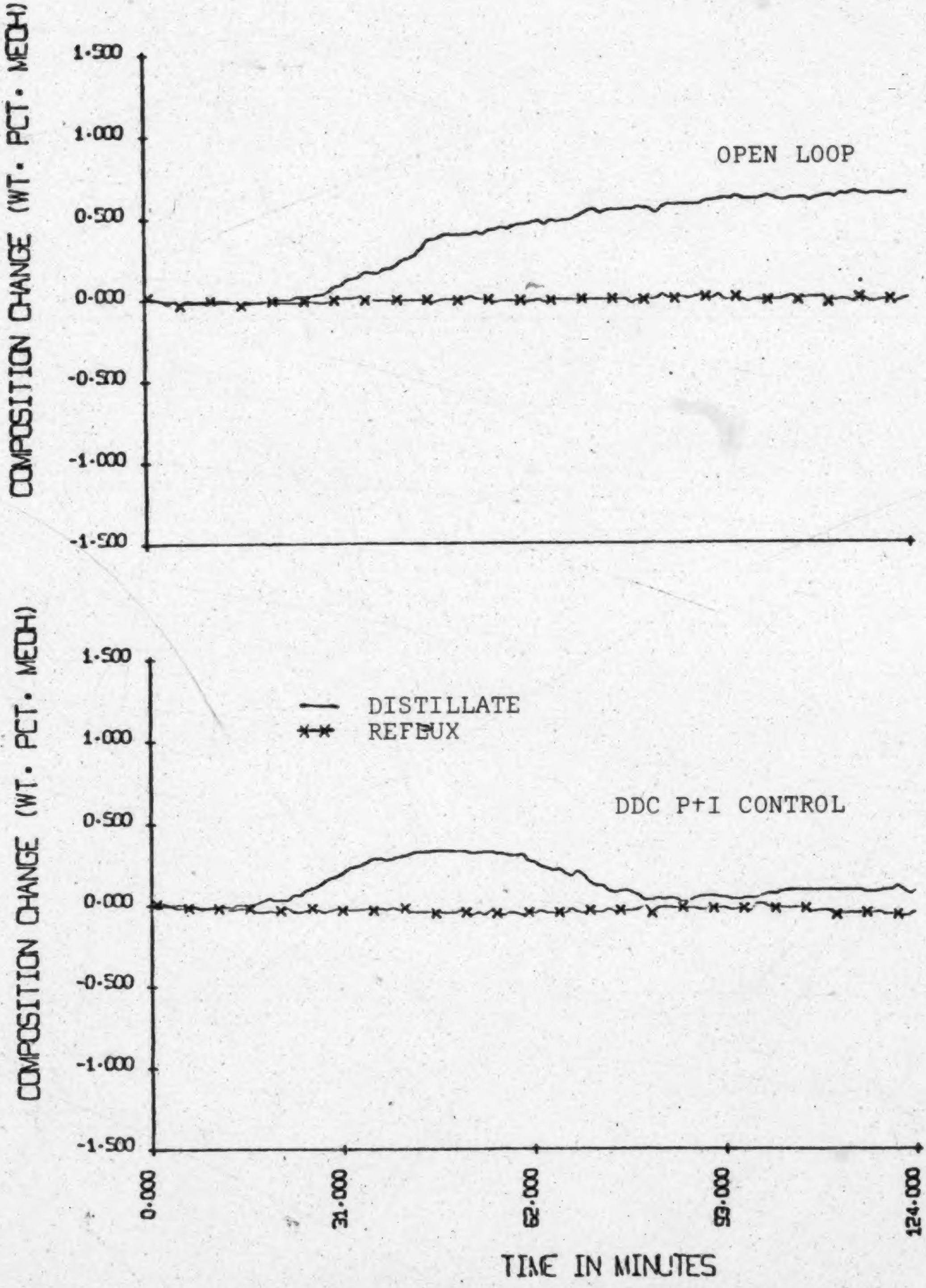


FIGURE 2.6: Experimental Response of Overhead Composition to a 10% Step Increase in Feed Flow using DDC.

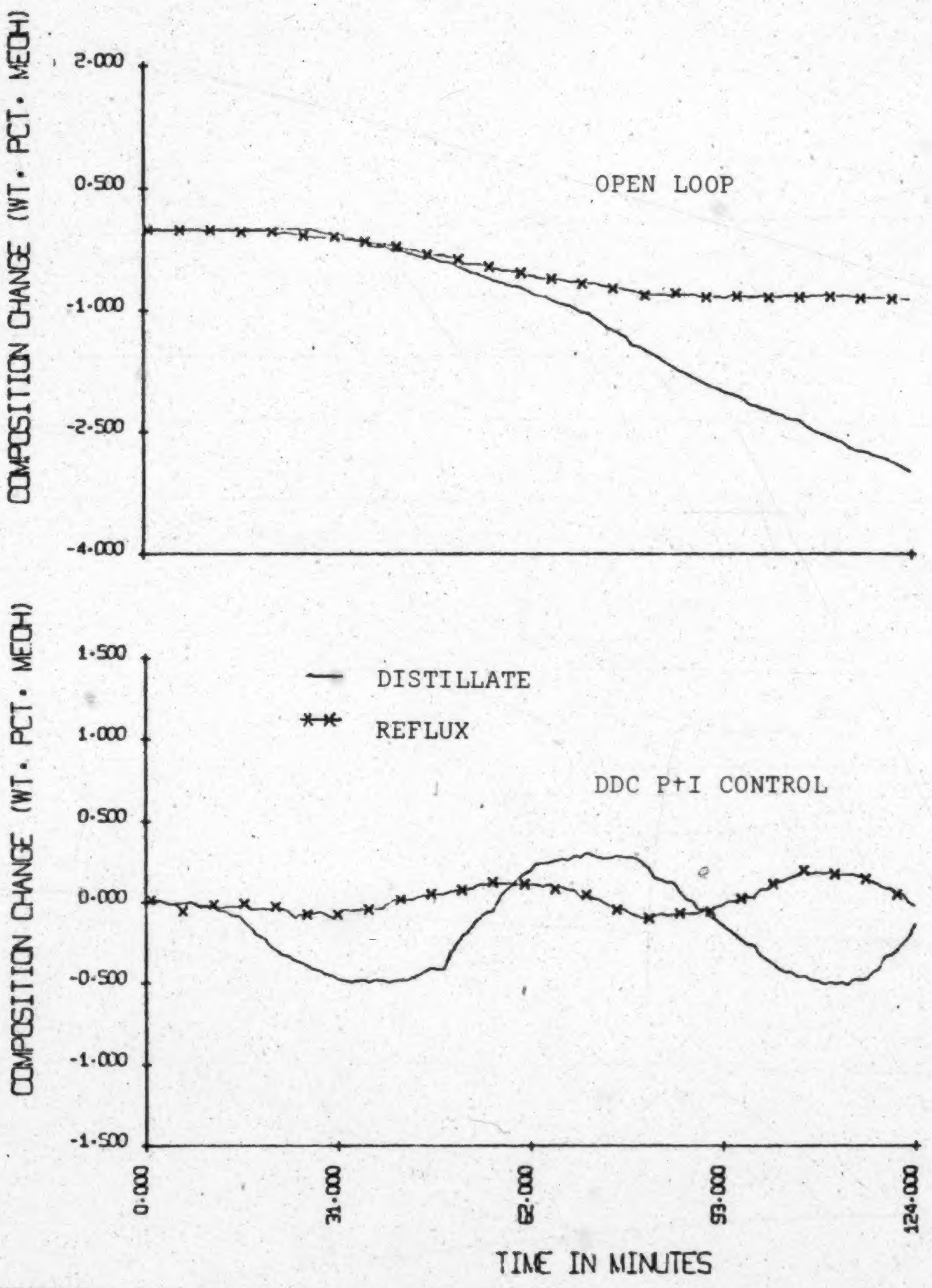


FIGURE 2.7: Experimental Response of Overhead Composition to a 10% Step Decrease in Feed using DDC.

linearity. In spite of the oscillatory response it is clear that the reflux case has better control behavior.

Figure 2.8 shows the response of overhead composition to a 71 Fahrenheit degree drop in feed temperature. The open loop response shows that the overhead composition is much more sensitive to a feed temperature change when reflux flow rather than distillate flow is the manipulative variable. This indicates that distillate flow should be used for composition control for such a disturbance, this is borne out by the closed loop response.

A further comparison to the previous work performed by Pacey is given in Figure 2.9 which shows responses of the overhead composition to a change in feed flow from 2.46 to 2.92 pounds per minute. It can be seen that the deviation of the composition is about twice as large when distillate flow is the manipulative variable compared to using reflux. However, by tuning the control the composition can be brought back to the setpoint in about the same period of time as for the reflux case but the IAE is still higher.

Since an overall control objective is ideally the control of both overhead and bottoms compositions, it is instructive to examine the deviation in bottoms composition as given in Tables 2.2 and 2.3. The bottoms composition analysis for the negative feed flow steps are bounded by zero percent so they are not meaningful. It should be noted that the gas chromatograph used for bottoms composition determination is not capable of detecting concentrations

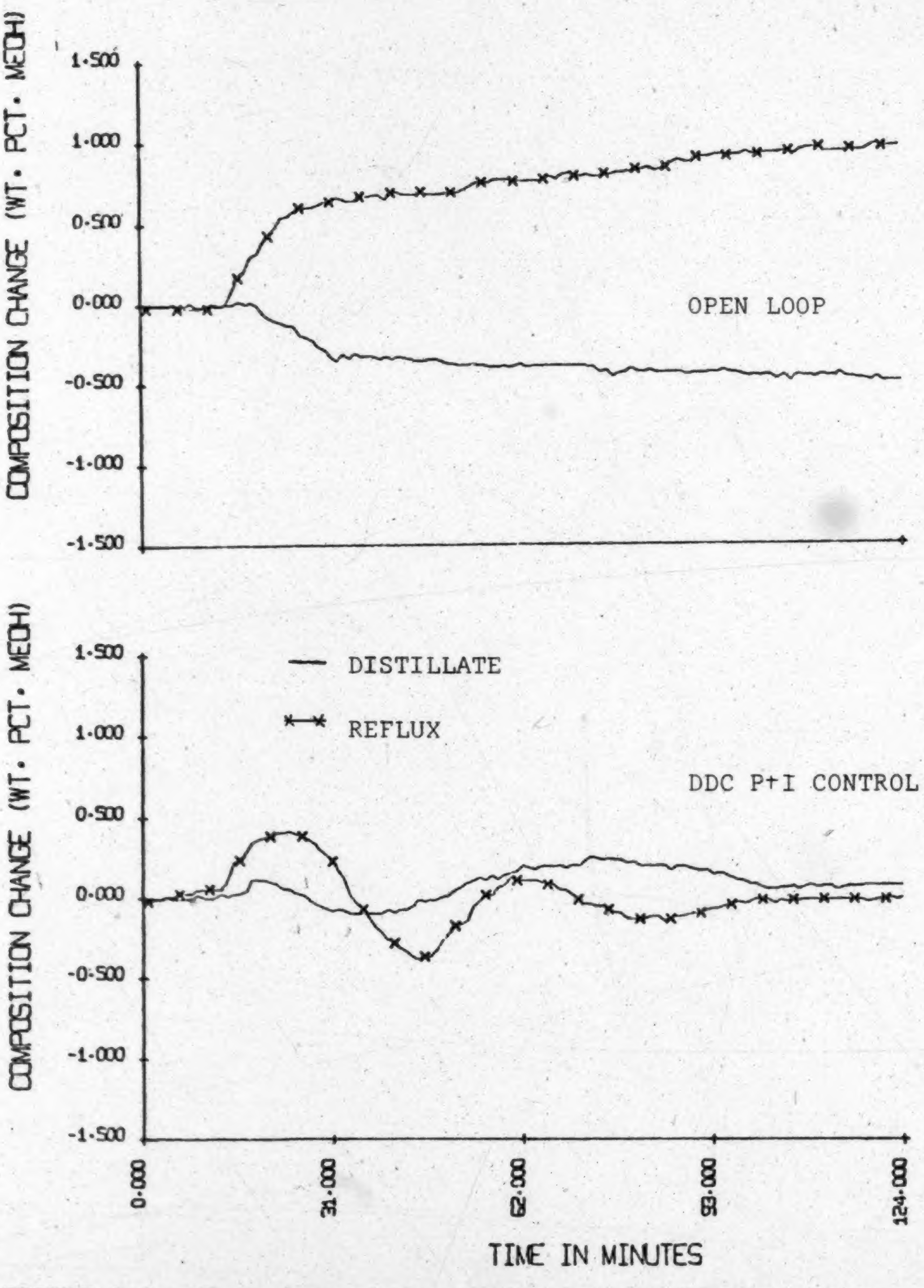


FIGURE 2.8: Experimental Response of Overhead Composition to a Step Decrease in Feed Temperature (161°-90°F.) using DDC.

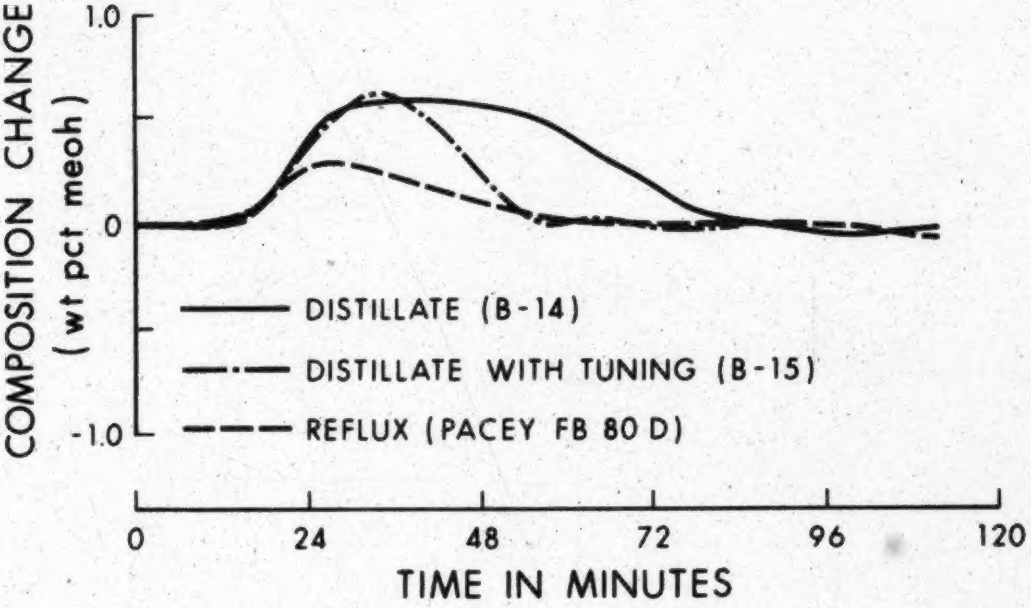


FIGURE 2.9: Comparison of Overhead Composition Response for an Increase in Feed Flow from 2.46 to 2.92 lb./min. for Control Using Distillate Flow to Control Using Reflux Flow.

below about one-half percent methanol by weight and results for compositions below one percent are not reliable. The deviation for positive feed flow changes and feed temperature drops indicate that the direct material balance control is better as it gives less interaction to the bottoms composition. The deviations in the bottoms composition recorded by Pacey are inexplicably far less than comparable runs shown here.

Also included in Table 2.3 are the initial and final steady states for the overhead composition and the deviation of the bottoms composition for open loop responses to steam flow changes (reboiler load change) when distillate and reflux flow are manipulated variables. The results indicate that the overhead is least affected by changes in steam rate when distillate flow is used for composition control rather than reflux flow.

2.4 Discussion of the Experimental Results

The results of the single variable control scheme study show evidence that if only the overhead composition is to be controlled it would be better to use the reflux flow. The runs using the analog controller generally indicate some improvement for the case using distillate flow to control the overhead composition but the improvement is not great enough to be conclusive. If, however, the DDC runs for feed flow steps are examined it can be seen that the difference is quite pronounced in favor of the reflux case. Since feed flow steps are the basic disturbance used on the column it would indicate that the reflux flow is the variable that should be used in order to get the best control.

The expansion of the problem to include control of both terminal compositions changes this conclusion. The fact that there is less interaction in both directions when distillate flow is used is strongly in favor of this configuration. It is because of this fact that the direct material balance control scheme utilizing distillate flow and reboiler load as manipulated variables to control overhead and bottoms compositions respectively was selected for use in the subsequent control study concerned with terminal composition control.

CHAPTER 3

MODELLING THE DISTILLATION COLUMN

3.1 Introduction

Some attempts have been made to model the University of Alberta pilot plant distillation column with varying success. Svrcek (48) derived a relatively complex model from theoretical considerations and experimental data. He was able to predict the experimental response of the column with reasonable accuracy. The studies that followed were concerned with fitting transfer function models to pulse test data. Berry (2) and Pacey (35) used first order plus time delay transfer function forms. They found that the system was non-linear to the extent that the models were only accurate for small excursions from the steady state. Chanh (7) fitted second order transfer functions with a time delay to pulse test data and again found the system quite non-linear.

McGinnis (28) modelled the column using the linearized liquid enthalpy relationship and a first order lag to represent the liquid dynamics for each tray. His assumption that the vapor flow change was negligible is questionable as he used reboiler load changes as one of his control variables in the subsequent control study. He overcame the problem by adding experimental values of the condenser load change to his runs and so obtained fairly good agreement between the predicted response from the model and the experimental response of the equipment. Another shortcoming of this model was that it gives good results only for conditions associated with high purity bottoms composition.

In view of the shortcomings of previous studies this work on modelling was undertaken to examine the system and the assumptions and to attempt to find a simple model with a wide range of use. The first item examined was whether liquid enthalpies or liquid compositions should be used to characterize the stages (states). Levy et al (22) applied modal analysis to the models resulting from using the energy and material balances for a binary distillation column separating an acetone-benzene mixture. They showed that the component mass balances were the best to use to characterize the system. Although this analysis is somewhat dependent on the physical system it is applicable to the University of Alberta distillation column.

In order to evaluate different models it is necessary to calculate the transient response of the states to some disturbance input. McClune and Gallier (27) who used the IBM program CSMP/360 (Continuous System Modelling Program/360) in designing a distillation column control system have discussed the advantages of using a digital simulation program. An improved version of this program CSMP III (57) was used in this study to calculate the predicted responses from the models for comparison to the experimental data.

Modelling of distillation processes has received much attention in the past such as the articles by Rosenbrock

(42,43,44). The literature dealing with modelling with respect to distillation has been categorized by McGinnis and by Levy et al showing the types of models used and the assumptions made. The reports are generally case studies because distillation processes cannot be generallized to any great extent. For that reason the emphasis in this study will be on obtaining a model for the University of Alberta distillation column; then the model can be examined for its general applicability.

3.2 Description of the Experimental Equipment

The University of Alberta distillation column is nine inches in diameter with glass walls to enable viewing of the process. It has eight bubble cap trays, a thermosyphon reboiler and a total condenser. A detailed description of the distillation column and the associated equipment is given by Svrcek (48) and by Pacey (35).

The column is operated with a feedrate of about 2.4 pounds per minute of methanol-water mixture containing 47 weight percent methanol. This is separated to about 2 percent in the bottoms and 97 percent in the overhead. A complete set of typical operating conditions are given in Appendix A. The overall control objective is to maintain both the overhead and bottoms compositions at some constant value.

The column has extensive instrumentation to enable a wide range of control studies to be undertaken. Temperature measurements are taken on all the trays and in the condenser and reboiler in the liquid phases. The feed, reflux, bottoms and distillate flowrates are measured as are their temperatures. Inlet and outlet temperatures and the flow of the steam and condenser cooling water are measured. All these measurements as well as the liquid compositions in the reboiler and condenser are transmitted to the IBM 1800 digital computer. The bottoms composition is measured by an industrial gas chromatograph controlled by the IBM 1800 on a cycle of approximately three minutes.

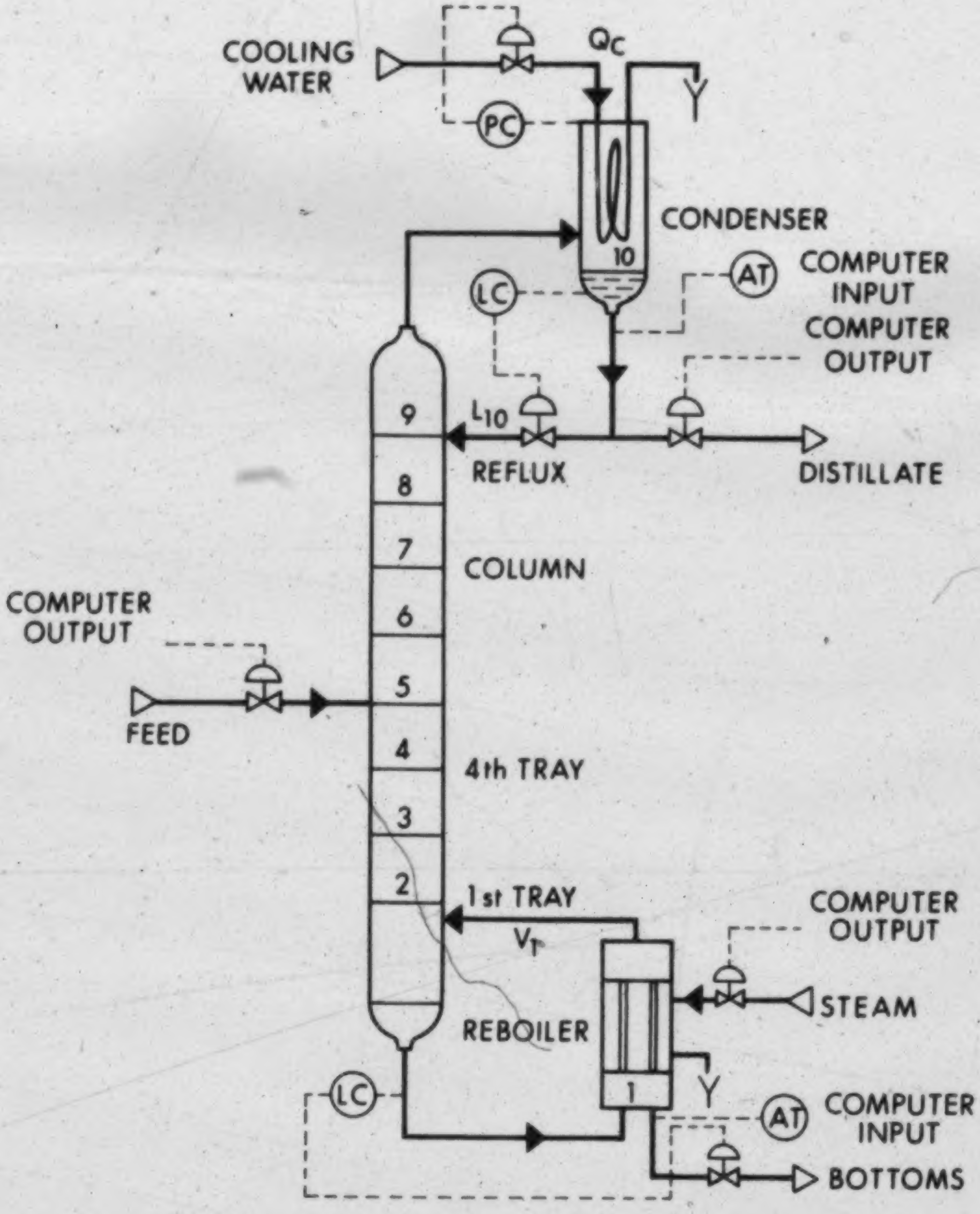
The column is outfitted with a complete set of analog controllers to control the flowrates as well as reboiler and condenser levels and the feed and reflux temperatures. The setpoints of the principle pneumatic flow controllers can be set from the IBM 1800 in a supervisory control mode.

A schematic diagram of the column giving details of the controls and measurements is given in Appendix A.

When the outputs from the computer are maintained constant the column control is said to be open loop even though the local variables such as level are controlled under analog controllers. A simple schematic of the column configuration is shown in Figure 3.1

3.3 Derivation of the Models

The usual approach to modelling a distillation column begins by considering the column to consist of a number of stages connected by mass and energy flows. Figure 3.2,



PC - PRESSURE CONTROLLER
LC - LEVEL CONTROLLER (*pneumatic*)
AT - COMPOSITION ANALYZER TRANSMITTER

FIGURE 3.1: Schematic of the Distillation Column Control Scheme

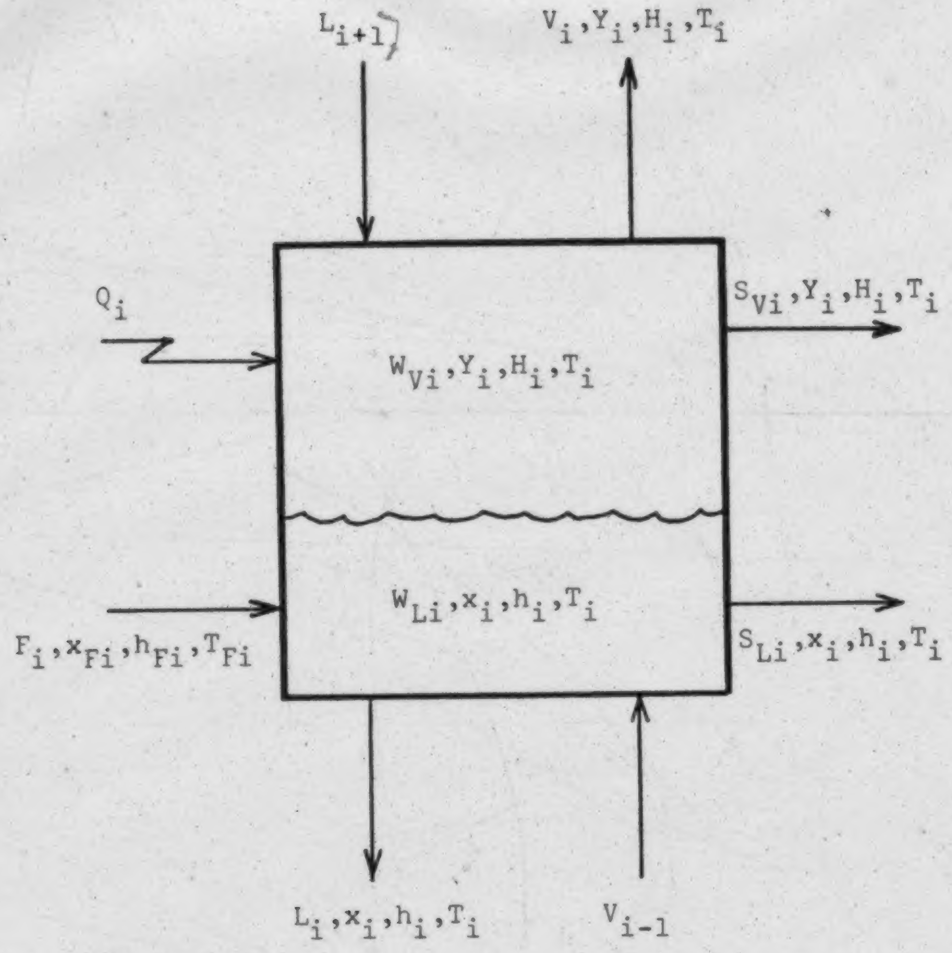


FIGURE 3.2: Representation of the i^{th} Stage.

which is a representation of a typical stage, shows the variables that will be used. The stage, used to represent a tray, reboiler or condenser will be considered to have two distinct phases, liquid and vapor. Each phase is assumed to be perfectly mixed. In addition other assumptions can be made about the stage such as a fixed relation between the physical properties of the liquid phase and those of the vapor phase.

For each stage material and energy balances can be written. For the time varying case the material balances can be stated as:

$$\frac{d}{dt}(W_{Li} + W_{Vi}) = L_{i+1} + V_{i-1} + F_i - (L_i + S_{Li}) - (V_i + S_{Vi}) \qquad 3.1$$

$$\frac{d}{dt}(x_i W_{Li} + Y_i W_{Vi}) = x_{i+1} L_{i+1} + Y_{i-1} V_{i-1} + x_{Fi} F_i - x_i (L_i + S_{Li}) - Y_i (V_i + S_{Vi}) \qquad 3.2$$

The energy balance is:

$$\frac{d}{dt}(h_i W_{Li} + H_i W_{Vi}) = h_{i+1} L_{i+1} + H_{i-1} V_{i-1} + h_{Fi} F_i - h_i (L_i + S_{Li}) - H_i (V_i + S_{Vi}) \qquad 3.3$$

Since the column contains eight trays plus reboiler and condenser there are ten stages in all. These balances yield thirty equations which contain 140 unknowns (fourteen per stage). In order to reduce the number of unknowns to thirty so that the equations can be solved some assumptions must be made.

Certain variables are classified as known inputs because they are measured or easily calculated and directly externally controllable. Included in this group are feed flow, enthalpy and composition, reboiler load, condenser load, reflux flow, distillate flow and bottoms flow. Thus the feed and sidestream drawoffs are known for each stage reducing the number of unknowns by fifty to ninety. If some assumption is made about heat loss for the stages such as constant heat loss the heat transfer for each stage is known leaving eighty unknowns.

It was stated previously that physical properties of the vapor could be assumed to have some functional relationship with those of the liquid phase. This means that vapor composition and enthalpy could be found from the liquid composition and enthalpy respectively thus reducing the unknowns by twenty to sixty.

Other assumptions can be made with respect to liquid and vapor dynamics. For example, assuming the vapor dynamics negligible reduces the unknowns to fifty because vapor holdups can be assumed to be the steady state values. On the other hand, liquid dynamics can be approximated by using weir equations or by assuming constant liquid holdup. This gives a relationship between holdup and liquid flow and reduces the unknowns by ten to forty.

The last ten unknowns may be eliminated by assuming some relationship for the vapor flowrate based on known inputs such as the reboiler load.

Even though the equations can now be solved (providing all the functional relationships assumed can be found) there is still the problem that on the actual column some variables such as liquid holdup are not measured. Control schemes such as optimal control require that all the "states" be available (measured). States in this case are the unknowns remaining in the stage equations. It might be possible to estimate some states by an estimation technique but in the case of liquid holdups it is doubtful if such a procedure would prove satisfactory. One way to eliminate the unwanted variables is to make more empirical assumptions. Another approach is to use numerical model reduction techniques such as Marshall's (26) modal reduction method or Anderson's (1) least squares reduction technique. Both approaches to eliminating states have disadvantages. The empirical approach is based on intuition and there criteria to determine whether one assumption is better than another and no guide as to what assumptions can justifiably be made. The modal and least squares reduction techniques require a linear system which generally means the system equations must be linearized. In addition there is the problem of choosing which states to retain.

Since the column is operated under pressure control via the condenser load it can be assumed that the pressure at each stage remains constant. As a result of this assumption it follows that if temperature (or liquid enthalpy) of a stage is known then the composition of the liquid phase can be established or knowing the composition the enthalpy can be calculated. Thus retaining both enthalpies and compositions as states is meaningless because they are related. The problem is in choosing which variable to retain. As no clear choice exists each was used to derive a model of the distillation column. In the derivations the liquid flow dynamics were neglected in order to simplify the model so that reduction was not necessary. Equations for liquid dynamics are presented for completeness.

3.3.1 Enthalpy Model

The simplest dynamic model that can predict the response of all ten stages must be of tenth order. Considering the pressure as constant and the liquid phase of each stage to be at the bubble point it is possible to specify the column compositions from temperatures or enthalpies. Possible shortcomings in this scheme include unaccounted for pressure deviations, non-saturated conditions, variable heat loss and incomplete or unreliable enthalpy data. Direct manipulation of the heat balance to control the primary variables namely the terminal compositions is possible using this model.

A summary of the various assumptions necessary to form a model containing, as the only unknowns, the liquid enthalpies on each stage are:

1. Constant pressure P_i = constant
   implies $x_i = f_1(h_i)$ and $h_i = f_2(T_i)$
2. Negligible liquid dynamics $\dot{W}_L = 0$
3. Negligible vapor dynamics $\dot{W}_V = 0$
4. Constant heat loss $q_i = 0$
5. Physical properties of the vapor are
   related to those of the liquid $Y_i = f_3(x_i)$
   $H_i = f_4(h_i)$

where f_1, f_2, f_3 and f_4 denote functional relationships and $i = 1,2,......,10$.

In addition to the above the vapor flow can be estimated from the heat input on the assumption that heat is transferred instantly up the column by changes in vapor flow. McGinnis (28) used the assumption that the vapor flowrate remained constant which is reasonable so long as the reboiler load is kept constant. However, as the reboiler load is one of the primary control variables it is considered a poor assumption. The actual relationship used for calculating the vapor rate from any stage includes the effect of heat losses from each tray and the effect of a non-saturated feed giving:

$$V_i = \left[Q_R + \sum_{j=1}^{i} (q_j + F_j\, \Delta h_{F_j})\right] \Big/ (H_i - h_i) \qquad 3.4$$

where Q_R - reboiler load = $Q_1 - q_1$

q_j - heat loss from each tray

Δh_{F_j} - the difference between the feed enthalpy and the saturated feed enthalpy.

In order to simplify the resulting equations a relation between the liquid and vapor enthalpy is needed. McGinnis used a linear relation based on the equilibrium data for the methanol-water system. His model was in linear perturbation form justifying his use of a linear relation. The intent in this work is to develop a non-linear model which has a wider range of valid operating conditions. The method chosen consisted of using pseudo-equilibrium data taken from steady state runs performed by McGinnis (28) and Svrcek (48).

The time varying enthalpy balance for a general stage is given by:

$$\begin{aligned} & h_i \dot{W}_{L_i} + \dot{h}_i W_{L_i} + H_i \dot{W}_{Vi} + \dot{H}_i W_{Vi} \\ & = V_{i-1} H_{i-1} + L_{i+1} h_{i+1} + F_i h_{Fi} \\ & \quad - (L_i + S_{Li}) h_i - (V_i + S_{Vi}) H_i + Q_i \end{aligned} \tag{3.5}$$

The terms containing the derivatives of the holdups can be neglected by considering liquid and vapor holdup to be constant. The term containing the vapor holdup was retained in spite of the fact that the magnitude of the vapor holdup is small because the derivative of the vapor enthalpy is relatively large. In order to retain that term the relationship between the derivative of the liquid and vapor enthalpy was linearized

$$\dot{H}_i = B_{1i} \dot{h}_i \qquad 3.6$$

where $$B_{1i} = \left. \frac{\delta H_i}{\delta h_i} \right|_{p=\text{constant}}$$

This can be justified by the fact that the small vapor holdup reduces the relative non-linear effect of this term and by the fact that this effects only the time response and not the steady state values.

The enthalpy balance can now be written as:

$$\dot{h}_i = [V_{i-1} H_{i-1} + L_{i+1} h_{i+1} + F_i h_{Fi} - (L_i + S_{Li}) h_i - (V_i + S_{Vi}) H_i + Q_{\underline{i}}] \Big/ [W_{Li} + B_{1i} W_{Vi}] \qquad 3.7$$

This equation for a general stage can now be modified to fit each of the stages in the actual column. This is carried out in Appendix B.

3.3.2 Concentration Model

The concentration model utilizes the liquid composition (concentration of methanol) to characterize stage behavior. This model has the advantage of explicitly using the terminal compositions as states.

The assumptions used for this model can be summarized as follows:

1. Constant pressure P_i = constant implying $x_i = f_6(T_i)$, $H_i = f_4(Y_i)$ $h_i = f_7(x_i)$
2. Negligible liquid dynamics $\dot{W}_{Li} = 0$
3. Negligible vapor dynamics $\dot{W}_{Vi} = 0$
4. Constant heat loss q_i = constant
5. Vapor composition is a function of liquid composition $Y_i = f_3(x_i)$

The same expression for vapor flow is used for the concentration model as was used for the enthalpy model.

The functional relationship between vapor and liquid compositions f_3, was found in the same manner as the relation between liquid and vapor enthalpy. The steady state data from Svrček (48) was smoothed to form a pseudo-equilibrium curve and represented by sufficient points to enable linear interpolation for simulation purposes.

The component balance can be written as:

$$\begin{aligned}\dot{W}_{Li}\, x_i + \dot{W}_{Vi}\, Y_i + W_{Li}\, \dot{x}_i + W_{Vi}\, \dot{Y}_i &\\ = L_{i+1}\, x_{i+1} + V_{i-1}\, Y_{i-1} - (L_i + S_{Li})\, x_i &\qquad 3.8\\ - (V_i + S_{Vi})\, Y_i + F_i\, x_{Fi} &\end{aligned}$$

The vapor holdups were considered negligible as they were less than a tenth of the liquid holdups so with some rearrangment equation 3.8 becomes:

$$\begin{aligned}\dot{x}_i = [&L_{i+1}\, x_{i+1} + V_{i-1}\, Y_{i-1} + F_i\, x_{Fi} \\ &- (L_i + S_{Li})\, x_i - (V_i + S_{Vi})\, Y_i] \big/ W_{Li}\end{aligned} \qquad 3.9$$

The modification of this equation to fit the actual stages is carried out in Appendix B.

3.4 Linearization of the Concentration Model

The control law chosen for this study is restricted to linear systems so the model was linearized by taking the linear terms of the Taylor Series expansion. This is necessary because the component balances contain product terms.

In addition it is necessary to have linear relationships for vapor flows and compositions. These relations are given in Appendix B.

The resulting equations for the system which appear in Appendix B are of the form

$$\dot{x}_i = f(\underline{z}, \underline{u}, \underline{d}) \qquad 3.10$$

where $\underline{z}$ - a vector of the states $[x_1, x_2, \ldots, x_{10}]$

$\underline{u}$ - a vector of control inputs $[D, Q_R]$

$\underline{d}$ - a vector of disturbance inputs $[F, x_F, h_F]$

This can be put in the form:

$$\dot{\underline{z}} = \underline{\underline{A}}\,\underline{z} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{C}}\,\underline{d} \qquad 3.11$$

where $\underline{\underline{A}}$, $\underline{\underline{B}}$ and $\underline{\underline{C}}$ - coefficient matrices

The coefficient matrices are constants for a particular initial steady state. A set of values for a typical steady state are given in Appendix A.

3.5 Results

The test program involved a series of experimental runs for different disturbances with the results being compared to the predicted response from the models. The models were tested for disturbances in feed flow, feed temperature, reboiler load and distillate flow.

The linear enthalpy model was tested first. The results from this were very poor. This was attributed to the high sensitivity of the enthalpy of the saturated solution versus concentration relationship. A plot of this relationship is given in Appendix A. This means that a very small error in the heat balance will make a large difference in the concentration at the higher methanol concentrations. No results from this model have been presented as the results were quite poor. This model was abandoned in favor of the concentration model.

The non-linear concentration model was next tested for its ability to accurately predict the response of the distillation column. The initial and final steady state values can be compared by examining the experimental values given in Table 3.1 and the values calculated from the model given in Table 3.2. Although these results were far better than those from the enthalpy model they are not as good as the various

TABLE 3.1

INITIAL AND FINAL STEADY STATE VALUES FROM EXPERIMENTAL OPEN LOOP RESPONSES

Run	Step Change	Bottoms Composition (Wt. Pct. MEOH)		Overhead Composition (Wt. Pct. MEOH)		Figure
		Initial	Final	Initial	Final	
B-20	10% F	1.6	6.5	96.0	96.6	3.3
B-21	-10% F	1.6	~ 0	96.4	*	
B-17	10% S	1.8	.4	96.5	96.4	3.4
B-22	-10% S	4.6	5.5	96.4	95.8	3.5
B-23	10% D	5.1	~ 0	96.6	*	
B-24	-10% D	2.0	5.2	95.8	96.6	3.7
B-25	5% D	6.2	3.8	97.2	96.7	3.6
B-26	$-T_F$ (161-142°F)	1.4	1.4	96.0	96.0	3.8

* Measurement went beyond instrument span.

TABLE 3.2

PREDICTED FINAL STEADY STATE VALUES FROM TENTH ORDER NONLINEAR CONCENTRATION MODEL FOR OPEN LOOP RESPONSES

Run	Step Disturbance	Steady State Bottoms Composition (Wt. Pct. MEOH)		Steady State Overhead Composition (Wt. Pct. MEOH)	
		Initial	Final	Initial	Final
D-1	10% F (2.4-2.64 lb/min)	1.451	7.730	96.47	96.81
D-2	10% Q_R (1782.1-1960.3 BTU/min)	4.887	4.696	96.86	97.09
D-3	5% D (1.058-1.113 lb/min)	6.340	2.812	97.00	96.75
D-4	-10% D (1.1-.99 lb/min)	2.232	9.203	96.70	97.10
D-5	$-T_F$ (161-141°F)	12.06	10.86	96.35	96.12
D-6	-10% Q_R (1770-1593 BTU/min)	3.340	3.770	96.76	96.24

results from the models presented by previous workers. This model is not as dependent on the accuracy of the input conditions as was the case of those of previous workers. The previous models were based on a specific set of operating conditions.

The linearized versions of this model were taken from the steady states calculated from the non-linear model using the input conditions of the experimental runs. The first three linear models were based on the initial conditions of the ten percent step increase in reboiler load, the five percent increase in distillate flow and the ten percent increase in feed flow. The final linearization was done around the final steady state of the ten percent step increase in feed flow. The steady state conditions used for linearization of the models are shown in Table 3.3. From Figure 3.3, which shows the predicted responses from the linearized models and the experimental data for a ten percent step disturbance in feed flow, it can be seen that the predicted response is highly dependent on the steady state around which the linearization is performed. Although this indicates that the system cannot be represented by a linear model for any sizeable deviation from the original steady state the linear model was used in the subsequent control study. The reason for this is simply that no method of incorporating the nonlinearities of the system in the type of control scheme being considered was found. If the system can be kept under tight control the deviation from the

TABLE 3.3

STEADY STATE CONDITIONS USED FOR LINEARIZATION

MODEL	1	2	3	4
Feed Flow (lb/min)	2.4	2.4	2.4	2.64
Reboiler Duty (BTU/min)	1782.1	1782.1	1783.0	1783.0
Distillate Flow (lb/min)	1.092	1.058	1.1	1.1
Feed Composition (Wt. Pct. MEOH)	46.74	46.31	44.86	44.86
Feed Temperature (°F)	161.0	161.0	161.0	161.0

initial steady state would be small so the linear model should be adequate.

The disturbance used to produce the response shown in Figure 3.3 was a ten percent increase in feed flow. The non-linear model gives the best prediction of the bottoms response. The overhead response is comparatively small at these conditions because the overhead composition is near the pinch region. The simulation predicts an even smaller overhead response. Examining the response reveals that the process has a time delay of about ten minutes. The actual system time constants lie between those of the linearized models but seem to be longer than that of the non-linear model.

The response for a negative change in feed flow is not plotted as the composition of both the overhead and bottoms streams went out of the range of the measuring systems.

The response of the system to a ten percent increase in steam flow (change in reboiler load) as shown in Figure 3.4 does not agree well with the predicted response from the concentration model. The overhead composition changed in the opposite direction from that predicted. From the response to a negative ten percent change in steam as shown in Figure 3.5 it can be seen that the change in overhead composition is in the same direction as for the positive step disturbance. No linear model can predict such behavior as the principle of superposition must hold. The physical

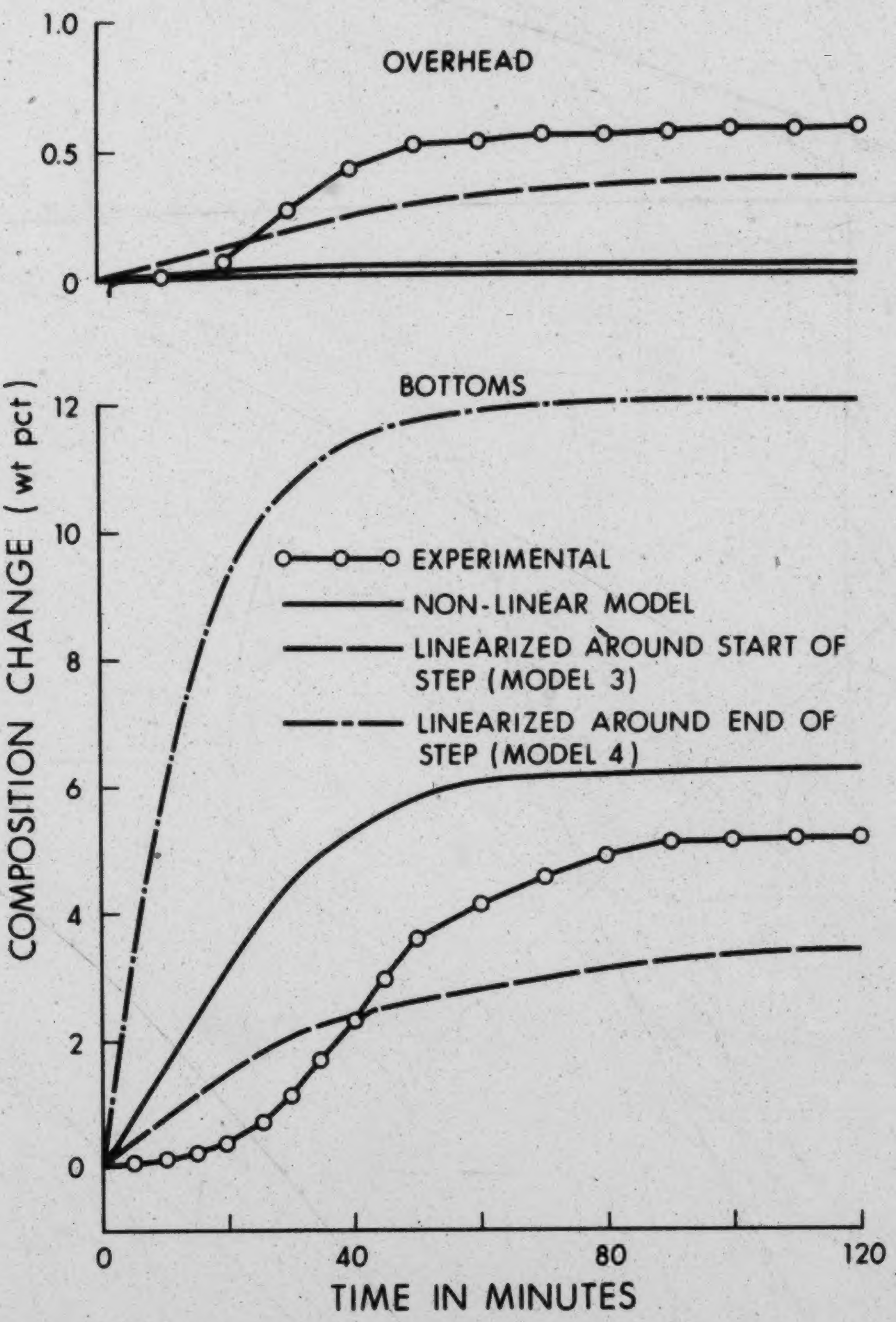


FIGURE 3.3: Comparison of Simulated Responses Predicted by Concentration Models and Experimental Response for a Ten Percent Step in Feed Flow (2.4-2.64 LB/MIN)

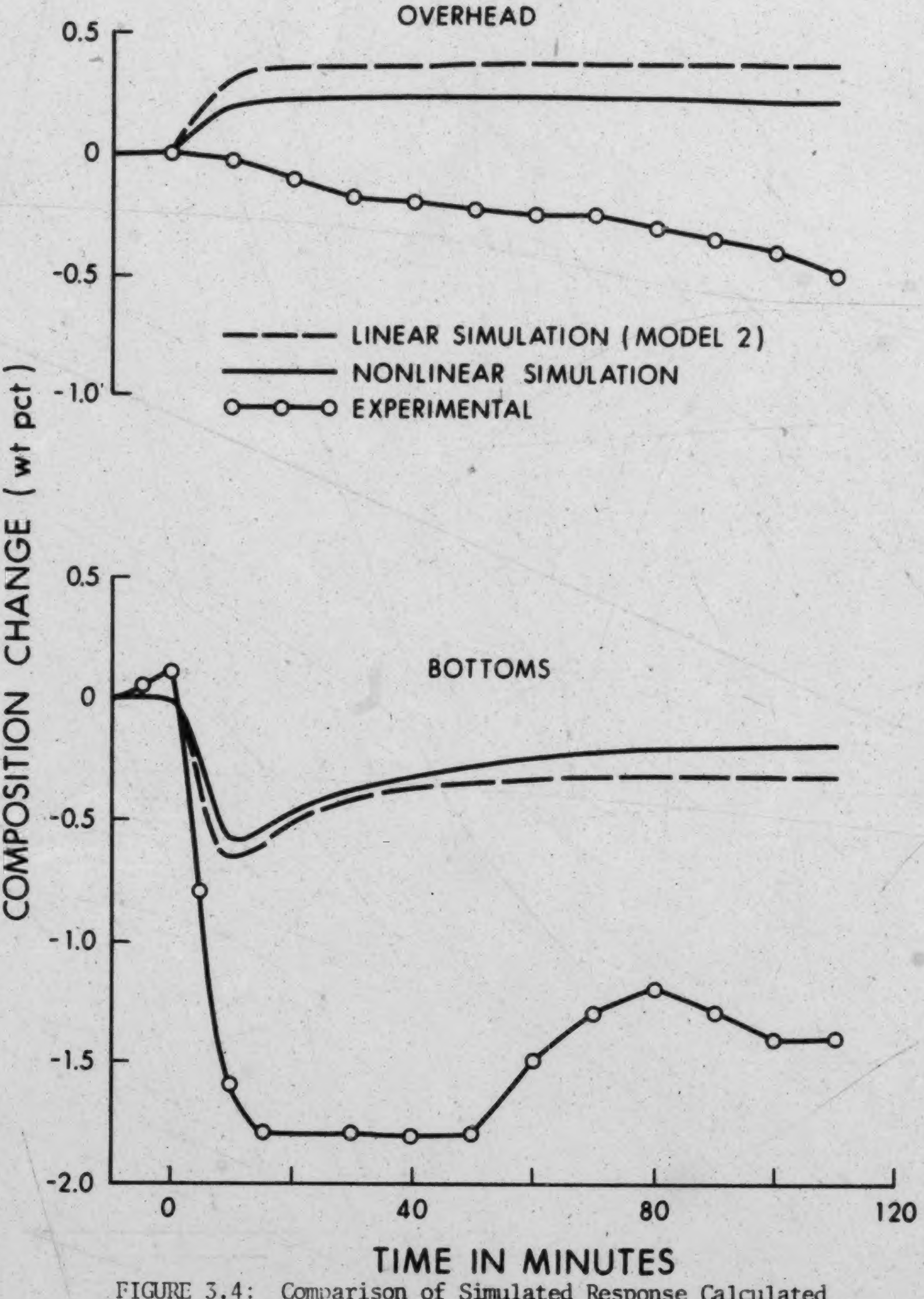


FIGURE 3.4: Comparison of Simulated Response Calculated From Linear Concentration Model to Experimental Response for a Ten Percent Step in Steam Flow to the Reboiler (1.85-2.035 LB/MIN)

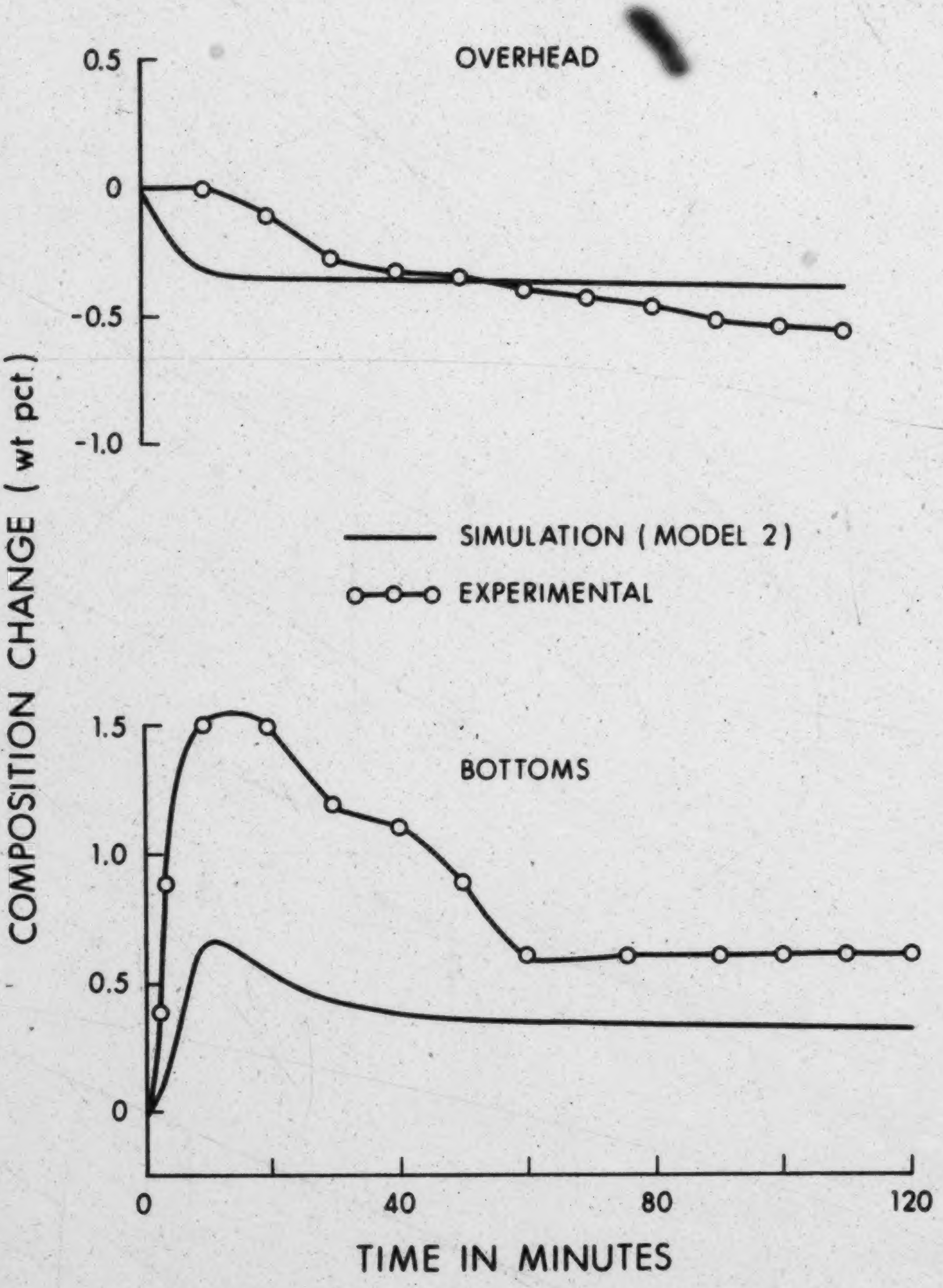


FIGURE 3.5: Comparison of Response Predicted by Linear Concentration Model to Experimental Response for a Ten Percent Step Decrease in Steam Flow (1.85-1.665 LB/MIN)

reason for this was deemed to be the change in efficiency due to the holdup change that accompanies a change in steam flow. The negative change in reboiler load gives better agreement. In comparing this response to that from the feed disturbance it can be seen that the time delay is no longer present and the response is fairly close in time for the simulation and the experimental results. However, the magnitude of the responses differ by a factor of two.

The step change disturbance of ten percent in the distillate flow resulted in the composition measurement instruments going off scale. A five percent increase was then carried out. The response of the terminal compositions along with the response predicted by the non-linear and linear models are shown in Figure 3.6. The final steady state agreement is fair but like the response to a feed flow step there is a time delay that has not been accounted for in the model. It is possible that this delay is due mainly to the liquid dynamic lags in the liquid flows on the trays. The negative change of ten percent in the distillate flow resulted in the response shown in Figure 3.7. The effect of an increase in plate holdup up to the point where flooding starts is evident here. The models predict concentrations that are lower for both overhead and bottoms up to flooding then overhead composition drops off.

The final disturbance is a change in feed temperature from 161°F to 142°F as shown in Figure 3.8. The response of the system is very small but appears to be in the correct

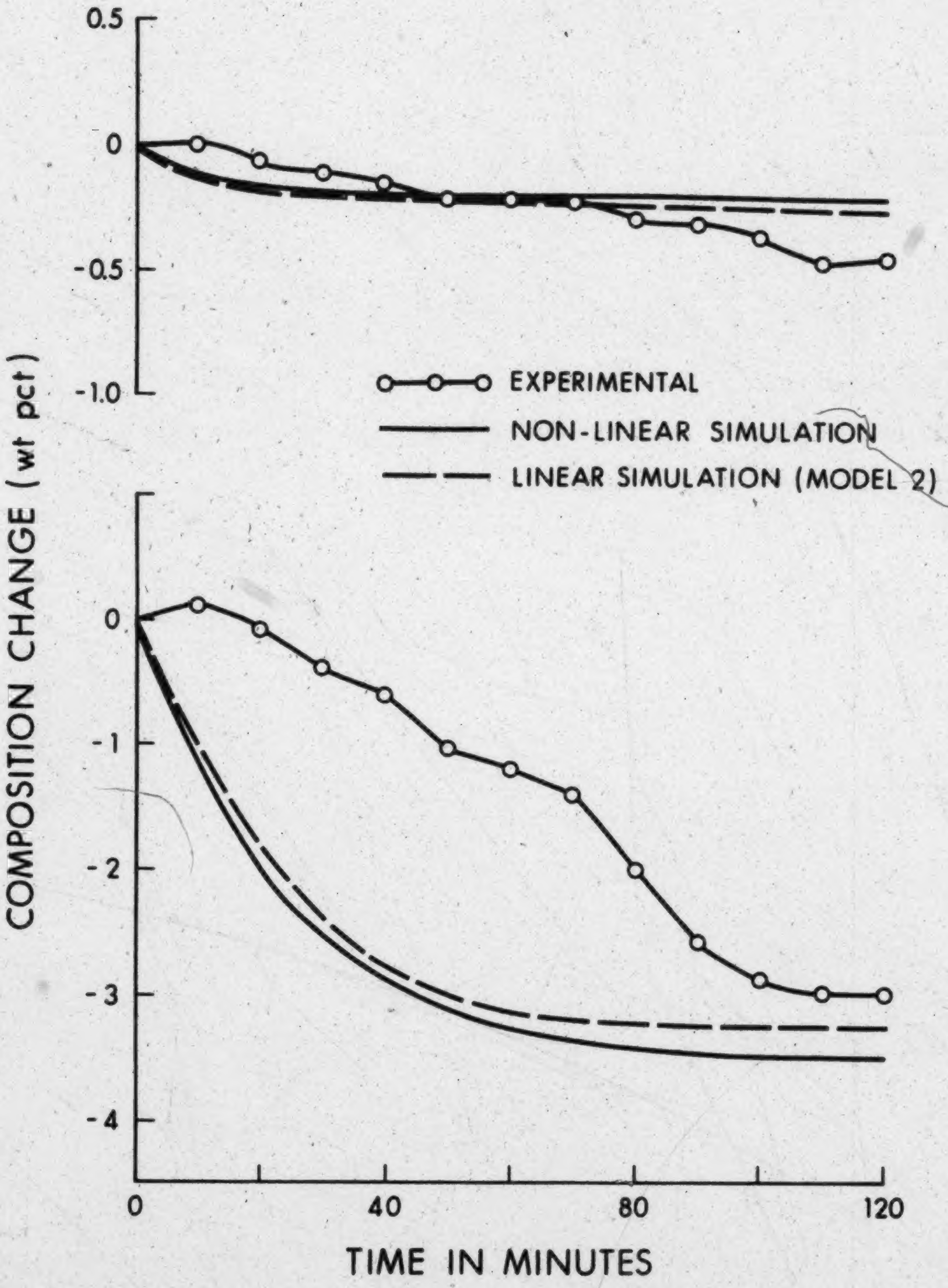


FIGURE 3.6: Comparison of Response Predicted by Linear Concentration Model to Experimental Response for a Five Percent Step in Distillate (1.058-1.108)

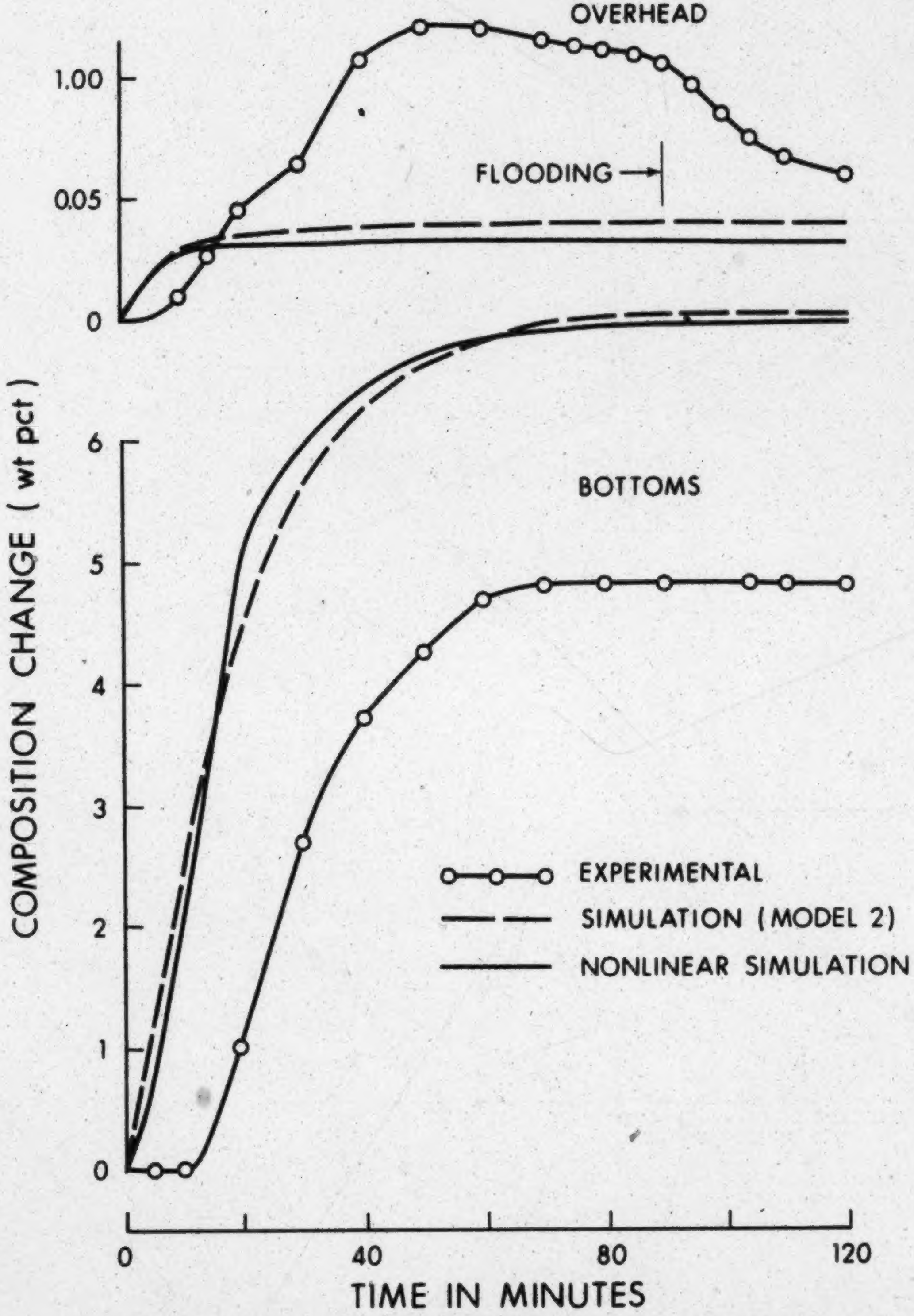


FIGURE 3.7: Comparison of Simulated Responses Calculated From Concentration Models to Experimental Response for a Ten Percent Step Decrease in Distillate Flow (1.1-.99 LB/MIN)

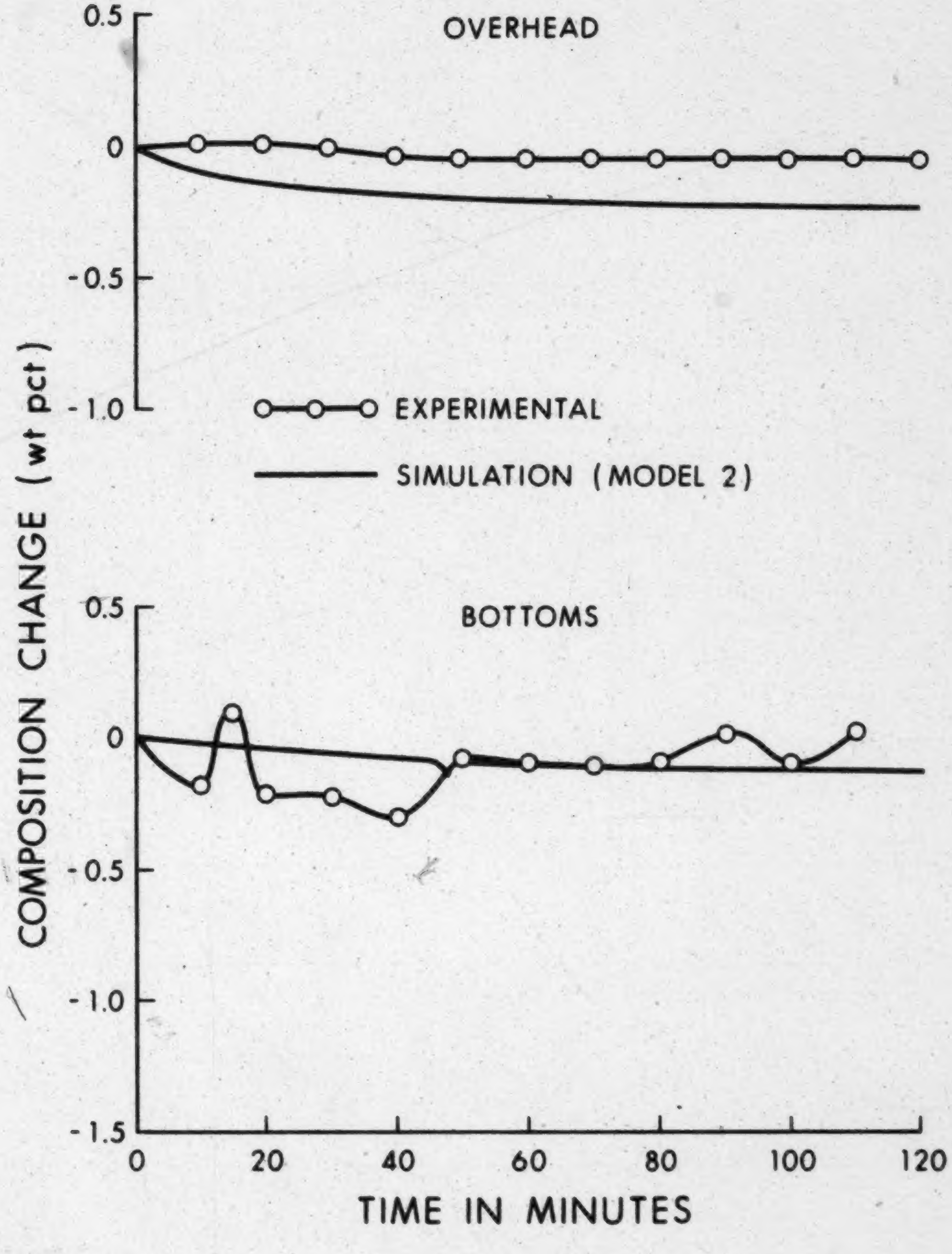


FIGURE 3.8: Comparison of Response Predicted by the Nonlinear Concentration Model to the Experimental Response for a Step Decrease in Feed Temperature (161-142°F)

direction. This shows the relative insensitivity of the system to mid-column energy changes.

It should be noted that the high noise level in the bottoms composition measurement necessitated some smoothing of the plots. This was done "by eye " on the unfiltered measurements. Improvement to the gas chromatograph measuring system or heavy filtering is necessary in order to obtain reasonable control. The predicted deviations are given in Table 3.4.

3.6 Discussion

The predictions of the model used here are not as satisfactory as those obtained previously by Svrcek, Pacey, Berry, Chanh and McGinnis. This means that agreement could be greatly improved by using more experimental data and by adding greater sophistication to the model by including such effects as the liquid dynamic lags and perhaps use an efficiency as a function of loading rather than the pseudo-equilibrium curve. The question still remains as to how much effort needs to be expended in model development in order to enable reasonable design of the control system. The answer to this question depends strongly on the design technique used to find a control scheme. All the previous workers found it necessary to tune the resulting control parameters when the scheme was implemented on the experimental equipment.

The fact that fairly large disturbances were used on a highly non-linear piece of equipment implies that the linear

TABLE 3.4

PREDICTED PERTURBATIONS FROM STEADY STATE FROM LINEARIZED TENTH ORDER CONCENTRATION MODEL FOR OPEN LOOP RESPONSES

Run	Model * Steady State	Step Disturbance	Perturbation in Bottoms Composition (Wt. Pct. MEOH)	Perturbation in Overhead Composition (Wt. Pct. MEOH)
E-1	1	F = .12 lb/min	3.78	.001
E-2	2	F = .12 lb/min	3.52	.017
E-3	3	F = .12 lb/min	3.59	.431
E-4	4	F = .12 lb/min	12.06	.006
E-5	1	D = .11 lb/min	-7.36	-.420
E-6	2	D = .11 lb/min	-7.09	-.390
E-7	3	D = .055 lb/min	-2.36	-.446
E-8	4	D = .11 lb/min	-6.06	-.442
E-9	1	Q_R = 178 BTU/min	-0.35	.419
E-10	2	Q_R = 178 BTU/min	-0.28	.357
E-11	3	Q_R = 178 BTU/min	-0.42	.496
E-12	4	Q_R = 178 BTU/min	1.43	.467
E-13	1	x_F = 46.7 Wt. Pct. MEOH	8.38	.199
E-14	2	x_F = 46.7 Wt. Pct. MEOH	8.15	.212
E-15	4	x_F = 46.7 Wt. Pct. MEOH	7.14	.218

* Steady State Conditions see Table 3.4.

model cannot be expected to give good results. In view of the phenomena that were assumed negligible the model gives reasonable representation of system behavior.

From a practical point of view the physical features such as measurement noise, accuracy and control action reliability could be more important than model accuracy in the final design steps. The measurement noise associated with the gas chromatograph which measures the bottoms composition is a significant shortcoming of the experimental system. This is one of the primary variables and the recorded measurement exhibits a noise level up to 100 percent or more of the actual measurement at the low compositions being measured. Other measurement problems include the effect of density changes on flow readings measured by a pressure drop across quadrant and square-edged orifices and the noise from the temperature measurements. In addition, control accuracy is affected by the fact that the control valves do not have positioners.

It appears that the assumptions made were at least partially effective. The vapor flowrate is somewhat affected by heat loss changes in the column. Since the insulating plexiglass was removed some variation here could be expected. The idea of pseudo-equilibrium has the advantage of enabling a wide range of operating conditions but it does not account for the change in efficiencies throughout the column affected by different loading so is limited in accuracy. The fact that the conditions used for the linearization were taken from the model rather than from experimental values leads to error but does give considerable ease in changing conditions. It would be possible to continuously calculate the controller based on the updated model using this method but for this approach to be advantageous the accuracy of the non-linear model would have to be improved.

CHAPTER 4

EXPERIMENTAL EVALUATION OF A CONTROL SCHEME

4.1 Introduction

For a distillation column it is possible to control the concentration of a component in both the overhead and bottoms streams. However, in practice, control is made difficult by the interaction present in the system and by sluggish response of the terminal compositions. The intent here is to present a possible solution to these problems by a modification of the control method proposed by Greenfield and Ward (13,14) and to apply the method using the concentration model derived in the previous chapter. The control behavior was first studied by simulation using the model before testing it on the actual equipment for experimental verification.

Previous workers at the University of Alberta have tested several schemes on the distillation column. Svrcek (48), who was concerned mainly with the dynamics of the system, tried only overhead composition control with a three mode feedback controller. Pacey (35) also controlled only the overhead composition but he showed that considerable improvement could be made by adding feedforward action. He made use of a method described by Luyben (25) to find feedforward control functions. Chanh (7) studied overhead composition control using a measurement of an intermediate tray temperature between the feed tray and the condenser. Use of an intermediate tray temperature, however, leads to final steady state offset.

The simultaneous control of both bottoms and overhead compositions was implemented by Berry (2). He showed that for two point composition control the interaction between the terminal compositions and the manipulated variables, reboiler load and reflux flow, degraded the control performance. He then tested two schemes to remove or diminish the interaction. The first proposed by Rijnsdorp and Van Kampen (40), used a ratio controller to control the reflux ratio while the ratio setpoint was adjusted by the overhead composition controller. The second scheme, non-interacting control, involved the addition of decoupling elements as outlined by Zalkind (55,56) and by Luyben (24). Unlike the first scheme, the second is based on the use of a transfer function model. The results indicate that the simpler method (reflux ratio scheme) gave results comparable to the non-interacting scheme but both schemes gave improvement over the case where there was no compensation for the interaction.

McGinnis (28), using a theoretical model, implemented the first multivariable control study of the column. The model was derived from non-steady state material and energy balances linearized around the steady state operating conditions. The model was put in the state space form with twenty states being the liquid enthalpies and the liquid flows from each stage. Since the control law of interest required that all states be measured it was found necessary to approximate the model using a subset of the states. This could have been accomplished by estimation techniques (21,23, 49) or by model reduction (1,26,52). He used Anderson's (1) method to reduce the model to second order retaining only the terminal compositions that were being measured and controlled. The reduction algorithm discretized the model enabling control to be carried out using a digital computer. McGinnis used a control scheme that contains both feedforward and feedback elements and does not require explicit decoupling as interactions are taken into account in the calculations. Although initial runs gave unsatisfactory oscillatory response due to the noise level in the measurement of the bottoms composition, McGinnis intuitively adjusted the values of elements in the control law to obtain satisfactory control. Comparison of the resulting control with that achieved by Berry, however, showed that the control behavior obtained in the previous study was superior. This was attributed to the fact that McGinnis did not account for time delays in his model.

Other control schemes have been developed which could be used to control the distillation column. Optimal control by definition should give the best control, however because of its dependence on the model and high gains often encountered optimal control may not the best choise. Hu and Ramirez (18) have presented control laws for the optimal control of distillation columns.

Rosenbrock (45) proposed a method of transforming the linear state space model into a non-interacting set of modes. This new system can then be used to calculate control without the need for decoupling. The dependence on having an accurate model is again the major shortcoming for this approach. The fact that the physical variables are not being dealt with directly adds a problem in visualizing the actual system. More recent treatments of modal control have been presented by Gould (12) and by Ellis and White (9,10,11).

The origin of the control technique used in this work, Greenfield and Ward's method which they call "structural analysis", makes use of the "structural" information available in the linearized material and energy balances. The structural information they use is contained in the state space model and is simply the relationships between the internal variables (or states) that are not given explicitly by transfer function models. The method is based on the "Invariance Principle" (17,36) and gives complete decoupling and feedforward control action with allowance for the addition of feedback loops. Bollinger and Lamb (3) proposed a similar control scheme based on a transfer function model form. The results of these two methods and one called V-form analysis proposed by Mesarovic (29) have been compared in a series of articles by Rajagopolan and Sheshadri (37,38, 39) for a continuous stirred tank reactor. It was found that when the chemical reaction in the reactor was second order the resulting control law from structural analysis required simple gains while the two transfer function techniques required lead-lag controllers. The simplicity of the controllers obtained by structural analysis is due to the extra relationships between the states that are available. It is on the basis of the simpler control law that the extra effort required to get the linear state space model can be justified.

The removal of the interactive effects by Greenfield and Ward's method includes both feedforward or input decoupling and feedback or output decoupling. The effectiveness of feedback decoupling has been discussed by Mitchell and Webb (30). They showed that adding decoupling does not always improve control. Niederlinski (32) presented some guidelines to help choose between decoupling and interactive control. It is easier to implement servo-control on decoupled loops. Interactive control may give better results for regulatory control but tuning of individual loops is difficult.

The final feature of Greenfield and Ward's method is the allowance made for the addition of feedback loops. The scheme compensates for interaction allowing the feedback loops to be treated as a single variable control problem. The advantage of this feature is apparent as it allows on-line tuning which permits the designer to compensate for model inadequacy.

4.2 Mathematical Development of the Control Scheme

The response of a process such as distillation can be represented by a set of first order non-linear differential equations. These equations can be linearized by retaining linear terms of a Taylor series expansion and the resulting set of equations can be put in the state space form. Using such a set of equations Greenfield and Ward have developed a scheme to decouple the system and introduce feedforward control. The motivation for using the state space model rather than the transfer function form lies in what Greenfield and Ward call the "structural" information. This is the equations giving the response of the internal "structural" variables or simply the states. These relationships simplify the derivation of an invariant control law. The "Invariance Principle" as discussed by Petrov (36) and by Haskins and Sliepcevich (17) is the principle behind Greenfield and Ward's control scheme.

The development shown here includes first the continuous system formulation of the control law for the case where some states are not measured and for the case where all states are measured or independently estimated. The discrete control law is formulated for the case where all states are measured since it is necessary that all states be available.

4.2.1 Continuous Formulation

The linear state space model can be expressed as

$$\dot{\underline{z}} = \underline{\underline{A}}\,\underline{z} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{C}}\,\underline{d} \qquad 4.1$$

where $\underline{z}$ - a vector of the states

$\underline{u}$ - a vector of the control inputs

$\underline{d}$ - a vector of the disturbance inputs

$\underline{\underline{A}},\underline{\underline{B}},\underline{\underline{C}}$ - model matrix, control matrix and disturbance matrix which are the coefficient matrices.

This model is in linear perturbation form so taking the Laplace transform gives

$$s\,\underline{Z} = \underline{\underline{A}}\,\underline{Z} + \underline{\underline{B}}\,\underline{U} + \underline{\underline{C}}\,\underline{D} \qquad 4.2$$

where $\underline{Z},\underline{U},\underline{D}$ = the Laplace transforms of $\underline{z},\underline{u}$ and $\underline{d}$ respectively

s = the independent variable (Laplace)

The states can be divided into:

$\underline{Z}_1$ - controlled states (measured or estimated)

$\underline{Z}_2$ - measured or estimated states not controlled

$\underline{Z}_3$ - unmeasured, uncontrolled and unestimated states

By partitioning the model, control and disturbance matrices, equation 4.2 can be written as:

$$s\begin{bmatrix}\underline{Z}_1\\ \underline{Z}_2\\ \underline{Z}_3\end{bmatrix} = \begin{bmatrix}\underline{\underline{A}}_1 & \underline{\underline{A}}_2 & \underline{\underline{A}}_3\\ \underline{\underline{A}}_4 & \underline{\underline{A}}_5 & \underline{\underline{A}}_6\\ \underline{\underline{A}}_7 & \underline{\underline{A}}_8 & \underline{\underline{A}}_9\end{bmatrix}\begin{bmatrix}\underline{Z}_1\\ \underline{Z}_2\\ \underline{Z}_3\end{bmatrix} + \begin{bmatrix}\underline{\underline{B}}_1\\ \underline{\underline{B}}_2\\ \underline{\underline{B}}_3\end{bmatrix}\underline{U} + \begin{bmatrix}\underline{\underline{C}}_1\\ \underline{\underline{C}}_2\\ \underline{\underline{C}}_3\end{bmatrix}\underline{D} \qquad 4.3$$

Note that it is assumed that the controlled states $\underline{Z}_1$ and the manipulated variables $\underline{U}$ are ordered such that the first manipulated input will be used for feedback control of the first state and the second for the second and so on to the last controlled state. Also, in this model it is necessary that the number of independent controlled outputs or states be less than or equal to the number of independent manipulated inputs (13) or:

$$\dim(\underline{Z}_1) < \dim(\underline{U})$$

where $\dim(\underline{Z}_1)$ - the dimension of the vector $\underline{Z}_1$

In the case where the system does not facilitate direct measurement of the states, it is generally possible to express the measured outputs as a linear combination of the states or:

$$\underline{y} = [\underline{\underline{H}}_1 \ \underline{\underline{H}}_2] \begin{bmatrix} \underline{Z}_1 \\ \underline{Z}_2 \end{bmatrix} \qquad 4.4$$

where $\underline{y}$ - output vector

$\underline{\underline{H}}_1, \underline{\underline{H}}_2$ - output matrices.

In this relationship it is necessary that

$$\dim(\underline{y}) = \dim(\underline{Z}_1) + \dim(\underline{Z}_2)$$

so that the matrix $[\underline{\underline{H}}_1 \ \underline{\underline{H}}_2]$ is square and:

$$\begin{bmatrix} \underline{Z}_1 \\ \underline{Z}_2 \end{bmatrix} = [\underline{\underline{H}}_1 \ \underline{\underline{H}}_2]^{-1} \underline{y} \qquad 4.5$$

The relationships given in equations 4.3 and 4.4 are represented in the signal flow graph given in Figure 4.1.

In addition to the dimensional restrictions already stated, two further requirements must be met in order to fulfil the invariance criterion:

1. Manipulated inputs are assumed unbounded under control demands.
2. No pure time delays are present in the system.

The first step in development of the control algorithm is the elimination of $\underline{Z}_3$ from the expression for s $\underline{Z}_1$. It follows from equation 4.3 that:

$$s\,\underline{Z}_1 = \underline{\underline{A}}_1\,\underline{Z}_1 + \underline{\underline{A}}_2\,\underline{Z}_2 + \underline{\underline{A}}_3\,\underline{Z}_3 + \underline{\underline{B}}_1\,\underline{U} + \underline{\underline{C}}_1\,\underline{D} \qquad 4.6$$

In order to eliminate $\underline{Z}_3$ it is necessary to solve for $\underline{Z}_3$ from the equation:

$$s\,\underline{Z}_3 = \underline{\underline{A}}_7\,\underline{Z}_1 + \underline{\underline{A}}_8\,\underline{Z}_2 + \underline{\underline{A}}_9\,\underline{Z}_3 + \underline{\underline{B}}_3\,\underline{U} + \underline{\underline{C}}_3\,\underline{D} \qquad 4.7$$

Solving for $\underline{Z}_3$ gives

$$\underline{Z}_3 = (s\,\underline{\underline{I}} - \underline{\underline{A}}_9)^{-1}\,(\underline{\underline{A}}_7\,\underline{Z}_1 + \underline{\underline{A}}_8\,\underline{Z}_2 + \underline{\underline{B}}_3\,\underline{U} + \underline{\underline{C}}_3\,\underline{D}) \qquad 4.8$$

where $\underline{\underline{I}}$ - identity matrix of appropriate dimension

Let

$$\underline{\underline{G}} = (s\,\underline{\underline{I}} - \underline{\underline{A}}_9)^{-1}$$

and substitute equation 4.8 into equation 4.6.

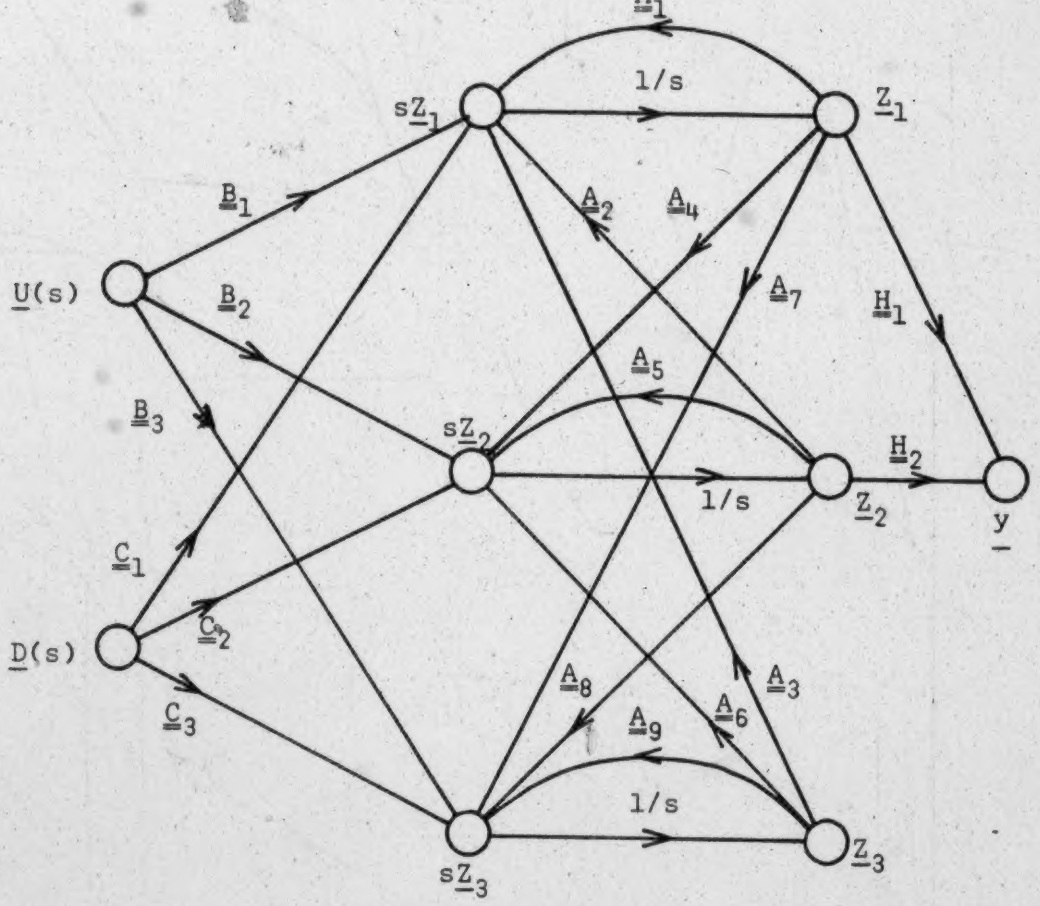

$\underline{\underline{A}}_1$
1/s
$s\underline{Z}_1$
$\underline{Z}_1$
$\underline{\underline{B}}_1$
$\underline{\underline{A}}_2$
$\underline{\underline{A}}_4$
$\underline{U}(s)$
$\underline{\underline{B}}_2$
$\underline{\underline{A}}_7$
$\underline{\underline{H}}_1$
$\underline{\underline{A}}_5$
$\underline{\underline{B}}_3$
$s\underline{Z}_2$
$\underline{\underline{H}}_2$
1/s
$\underline{Z}_2$
$\underline{y}$
$\underline{\underline{C}}_1$
$\underline{\underline{C}}_2$
$\underline{D}(s)$
$\underline{\underline{A}}_8$
$\underline{\underline{A}}_3$
$\underline{\underline{C}}_3$
$\underline{\underline{A}}_9$
$\underline{\underline{A}}_6$
1/s
$s\underline{Z}_3$
$\underline{Z}_3$

FIGURE 4.1: Signal Flow Graph of the Model Equations.

$$s\,\underline{Z}_1 = (\underline{\underline{A}}_1 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{A}}_7)\underline{Z}_1 + (\underline{\underline{A}}_2 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{A}}_8)\underline{Z}_2 + (\underline{\underline{B}}_1 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{B}}_3)\underline{U} + (\underline{\underline{C}}_1 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{C}}_3)\underline{D} \qquad 4.9$$

Let

$$\begin{aligned} \underline{\underline{L}} &= \underline{\underline{A}}_1 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{A}}_7 \\ \underline{\underline{M}} &= \underline{\underline{A}}_2 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{A}}_8 \\ \underline{\underline{P}} &= \underline{\underline{B}}_1 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{B}}_3 \\ \underline{\underline{Q}} &= \underline{\underline{C}}_1 + \underline{\underline{A}}_3\,\underline{\underline{G}}\,\underline{\underline{C}}_3 \end{aligned} \qquad 4.10$$

Separate matrix $\underline{\underline{L}}$ into two matrices such that one matrix, $\underline{\underline{L}}_d$, is diagonal containing the main diagonal of $\underline{\underline{L}}$ and the other, $\underline{\underline{L}}_p$, consists of the original matrix with the diagonal set to zero such that

$$\underline{\underline{L}}_d + \underline{\underline{L}}_p = \underline{\underline{L}} \qquad 4.11$$

Substituting relations 4.10 and 4.11 into 4.9 gives

$$s\,\underline{Z}_1 = (\underline{\underline{L}}_d + \underline{\underline{L}}_p)\underline{Z}_1 + \underline{\underline{M}}\,\underline{Z}_2 + \underline{\underline{P}}\,\underline{U} + \underline{\underline{Q}}\,\underline{D} \qquad 4.12$$

Rearranging gives

$$\underline{Z}_1 = (s\,\underline{\underline{I}} - \underline{\underline{L}}_d)^{-1}\,(\underline{\underline{L}}_p\,\underline{Z}_1 + \underline{\underline{M}}\,\underline{Z}_2 + \underline{\underline{P}}\,\underline{U} + \underline{\underline{Q}}\,\underline{D}) \qquad 4.13$$

This represents a set of linear equations of the form

$$Z_i = f_i\,(\underline{Z}_1'', \underline{Z}_2, \underline{U}, \underline{D}) \qquad 4.14$$

where Z_i - the ith element of $\underline{Z}_1$

$\underline{Z}_1''$ - vector $\underline{Z}_1$ with element i deleted

For invariance $Z_i = 0$ in equation 4.14 or equation 4.13 becomes

$$\underline{\underline{0}} = (s\,\underline{\underline{I}} - \underline{\underline{L}}_d)^{-1}\,(\underline{\underline{L}}_p\,\underline{Z}_1 + \underline{\underline{M}}\,\underline{Z}_2 + \underline{\underline{P}}\,\underline{U} + \underline{\underline{Q}}\,\underline{D}) \qquad 4.15$$

where $\underline{\underline{0}}$ - null matrix

which will be satisfied if

$$\underline{\underline{L}}_p\,\underline{Z}_1 + \underline{\underline{M}}\,\underline{Z}_2 + \underline{\underline{P}}\,\underline{U} + \underline{\underline{Q}}\,\underline{D} = \underline{\underline{0}} \qquad 4.16$$

Separate $\underline{\underline{P}}$ into two matrices as was done with $\underline{\underline{L}}$ to get

$$\underline{\underline{P}}_d + \underline{\underline{P}}_p = \underline{\underline{P}} \qquad 4.17$$

and substituting into equation 4.16 gives

$$\underline{\underline{L}}_p\,\underline{Z}_1 + \underline{\underline{M}}\,\underline{Z}_2 + \underline{\underline{P}}_d\,\underline{U} + \underline{\underline{P}}_p\,\underline{U} + \underline{\underline{Q}}\,\underline{D} = \underline{\underline{0}} \qquad 4.18$$

which upon rearrangement yields

$$\underline{U} = -\underline{\underline{P}}_d^{-1}\,(\underline{\underline{L}}_p\,\underline{Z}_1 + \underline{\underline{M}}\,\underline{Z}_2 + \underline{\underline{P}}_p\,\underline{U} + \underline{\underline{Q}}\,\underline{D}) \qquad 4.19$$

which is of the form

$$U_i = f_i\,(\underline{Z}_1'', \underline{Z}_2, \underline{U}'', \underline{D}) \qquad 4.19a$$

where U_i - element of $\underline{U}$

$\underline{U}''$ - vector $\underline{U}$ with element i deleted

Equation 4.19 gives the relation used for feedforward plus decoupling control as obtained by Greenfield and Ward. Substituting equation 4.5 gives

$$\underline{U} = -\underline{\underline{P}}_d^{-1}\,([\underline{\underline{L}}_p\;\underline{\underline{M}}][\underline{\underline{H}}_1\;\underline{\underline{H}}_2]^{-1}\,\underline{y} + \underline{\underline{P}}_p\,\underline{U} + \underline{\underline{Q}}\,\underline{D}) \qquad 4.20$$

The control matrices thus are:

$$\begin{aligned}\underline{\underline{K}}_{FF} &= -\underline{\underline{P}}_d^{-1}\,\underline{\underline{Q}}\\ \underline{\underline{K}}_{DC} &= -\underline{\underline{P}}_d^{-1}[\underline{\underline{L}}_p\ \underline{\underline{M}}][\underline{\underline{H}}_1\ \underline{\underline{H}}_2]^{-1}\\ \underline{\underline{K}}_{UC} &= -\underline{\underline{P}}_d^{-1}\underline{\underline{P}}_p\end{aligned} \qquad 4.21$$

where $\underline{\underline{K}}_{FF}$, $\underline{\underline{K}}_{DC}$, $\underline{\underline{K}}_{UC}$ - control matrices - feedforward, output decoupling, input decoupling.

The control system is represented with a signal flow graph in Figure 4.2.

At this point feedback control action can be included in the control function:

Let $\underline{V}$ - the feedback portion added $V_i - f_i\ (Z_i)$

(for P + I feedback $V_i = K_p Z_i + K_I \int_{t=0}^{T} Z_i dt$ at t=T)

thus adding $\underline{V}$ to equation 4.20 gives

$$\underline{U} = -\underline{\underline{P}}_d^{-1}\ (\underline{\underline{L}}_p\ \underline{Z}_1 + \underline{\underline{M}}\ \underline{Z}_2 + \underline{\underline{P}}_p\ \underline{U} + \underline{\underline{Q}}\ \underline{D}) + \underline{V} \qquad 4.22$$

For the case where all states are measured equation 4.2 can be written as

$$s\begin{bmatrix}\underline{Z}_1\\ \underline{Z}_2\end{bmatrix} = \begin{bmatrix}\underline{\underline{A}}_1 & \underline{\underline{A}}_2\\ \underline{\underline{A}}_3 & \underline{\underline{A}}_4\end{bmatrix}\begin{bmatrix}\underline{Z}_1\\ \underline{Z}_2\end{bmatrix} + \begin{bmatrix}\underline{\underline{B}}_1\\ \underline{\underline{B}}_2\end{bmatrix}\underline{U} + \begin{bmatrix}\underline{\underline{C}}_1\\ \underline{\underline{C}}_2\end{bmatrix}\underline{D} \qquad 4.23$$

where the states are as defined previously.

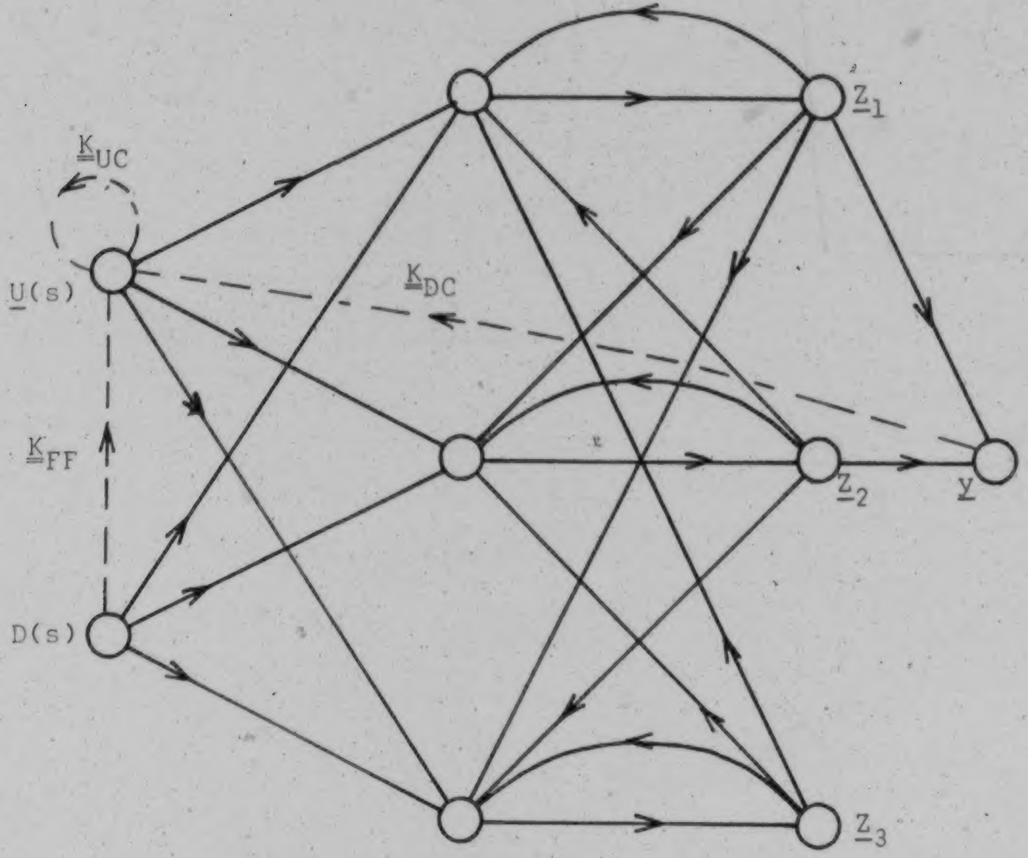


FIGURE 4.2: Signal Flow Graph of the System with Feedforward and Decoupling Control.

It thus follows that $s\,\underline{Z}_1$ is

$$s\,\underline{Z}_1 = \underline{\underline{A}}_1\,\underline{Z}_1 + \underline{\underline{A}}_2\,\underline{Z}_2 + \underline{\underline{B}}_1\,\underline{U} + \underline{\underline{C}}_1\,\underline{D} \qquad 4.24$$

Forming the diagonal matrix $\underline{\underline{A}}_d$ and the matrix $\underline{\underline{A}}_p$ consisting of $\underline{\underline{A}}_1$ with the main diagonal set to zero such that

$$\underline{\underline{A}}_p + \underline{\underline{A}}_d = \underline{\underline{A}}_1 \qquad 4.25$$

Substituting into equation 4.24 and rearranging for $\underline{Z}_1$ gives

$$\underline{Z}_1 = (s\,\underline{\underline{I}} - \underline{\underline{A}}_d)^{-1}\,(\underline{\underline{A}}_p\,\underline{Z}_1 + \underline{\underline{A}}_2\,\underline{Z}_2 + \underline{\underline{B}}_1\,\underline{U} + \underline{\underline{C}}_1\,\underline{D}) \qquad 4.26$$

As before invariance holds if

$$\underline{\underline{A}}_p\,\underline{Z}_1 + \underline{\underline{A}}_2\,\underline{Z}_2 + \underline{\underline{B}}_1\,\underline{U} + \underline{\underline{C}}_1\,\underline{D} = \underline{\underline{0}} \qquad 4.27$$

Separation of $\underline{\underline{B}}_1$ into a diagonal matrix $\underline{\underline{B}}_d$ and a matrix with a zero main diagonal such that

$$\underline{\underline{B}}_p + \underline{\underline{B}}_d = \underline{\underline{B}}_1 \qquad 4.28$$

and substitution into equation 4.27 and rearranging yields:

$$\underline{U} = \underline{\underline{B}}_d^{-1}\,(\underline{\underline{A}}_p\,\underline{Z}_1 + \underline{\underline{A}}_2\,\underline{Z}_2 + \underline{\underline{B}}_p\,\underline{U} + \underline{\underline{C}}_1\,\underline{D}) \qquad 4.29$$

This is the control function in the form presented by Greenfield and Ward. Unlike the previous case where some states were eliminated algebraically this result has constant control matrices. An equivalent expression for $\underline{U}$ is obtained by solving equation 4.27

$$\underline{U} = -\underline{\underline{B}}_1^{-1}\,(\underline{\underline{A}}_p\,\underline{Z}_1 + \underline{\underline{A}}_2\,\underline{Z}_2 + \underline{\underline{C}}_1\,\underline{D}) \qquad 4.30$$

Expressing equation 4.30 in terms of the output gives:

$$\underline{U} = -\underline{\underline{B}}_1^{-1} \left([\underline{\underline{A}}_p \ \underline{\underline{A}}_2][\underline{\underline{H}}_1 \ \underline{\underline{H}}_2]^{-1} \underline{y} + \underline{\underline{C}}_1' \ \underline{D} \right) \qquad 4.31$$

giving control equations

$$\underline{\underline{K}}_{FF} = -\underline{\underline{B}}_1^{-1} \underline{\underline{C}}_1$$

$$\underline{\underline{K}}_{DC} = -\underline{\underline{B}}_1^{-1} [\underline{\underline{A}}_p \ \underline{\underline{A}}_2][\underline{\underline{H}}_1 \ \underline{\underline{H}}_2]^{-1} \qquad 4.31a$$

The difference between these results is that equation 4.30 unlike equation 4.29 does not contain explicit decoupling of the control inputs. This becomes important when the feedback portion of the control is added. Adding feedback control to equation 4.29 gives:

$$\underline{U} = -\underline{\underline{B}}_d^{-1} \left(\underline{\underline{A}}_p \underline{Z}_1 + \underline{\underline{A}}_2 \underline{Z}_2 + \underline{\underline{B}}_p \underline{U} + \underline{\underline{C}}_1 \underline{D} \right) + \underline{V} \qquad 4.32$$

while inclusion with equation 4.30 yields

$$\underline{U} = -\underline{\underline{B}}_1^{-1} \left(\underline{\underline{A}}_p \underline{Z}_1 + \underline{\underline{A}}_2 \underline{Z}_2 + \underline{\underline{C}}_1 \underline{D} \right) + \underline{V} \qquad 4.33$$

To illustrate the difference a third order example is presented in Appendix C. The control function as given in equation 4.31 is illustrated in Figure 4.3. The advantage of this form is that when control is carried out by a digital computer it is unnecessary to solve the set of equations for each control output by numerical means as would be the case using equation 4.29 (or 4.32) because $\underline{U}$ is given explicitly.

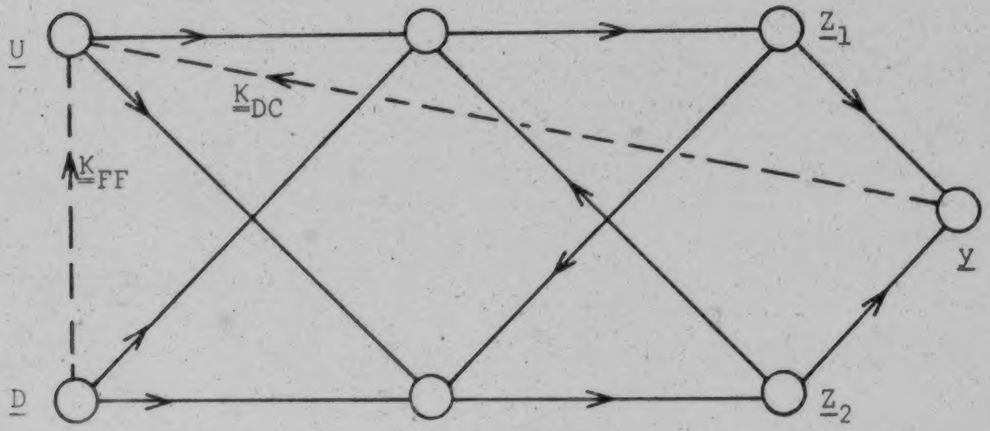


FIGURE 4.3: Signal Flow Graph of System Incorporating Simplified Control Scheme.

4.2.2 Discrete Formulation

The previous formulation of the control function can also be adapted for the discrete version of the linear state space model although an additional restriction must be met. In the discrete formulation the elimination of unmeasured states from the model cannot be done directly so that states that cannot be measured or estimated must be removed by some method such as modal reduction.

The discrete model equivalent to the continuous linear state space model used in the previous section can be expressed as

$$\underline{z}[(k+1)T] = \underline{\underline{\phi}}(T)\ \underline{z}(kT) + \underline{\underline{\beta}}(T)\ \underline{u}(kT) + \underline{\underline{\Delta}}(T)\ \underline{d}(kT) \qquad 4.34$$

where T - the discrete time interval

k - the integer number of intervals

$\underline{z}(kT)$ - vector of states at time kT

$\underline{u}(kT)$ - vector of control inputs at time kT

$\underline{d}(kT)$ - vector of disturbance inputs at time kT

$\underline{\underline{\phi}}(T)$ - fundamental matrix

$\underline{\underline{\beta}}(T)$ - control matrix

$\underline{\underline{\Delta}}(T)$ - disturbance matrix

For simplicity the symbol for the discrete time interval T will be omitted from the following derivation.

Dividing the states into those controlled $\underline{z}_1$ and those not controlled $\underline{z}_2$ allows the model to be given as:

$$\begin{bmatrix} \underline{z}_1(k+1) \\ \underline{z}_2(k+1) \end{bmatrix} = \begin{bmatrix} \underline{\Phi}_1 & \underline{\Phi}_2 \\ \underline{\Phi}_3 & \underline{\Phi}_4 \end{bmatrix} \begin{bmatrix} \underline{z}_1(k) \\ \underline{z}_2(k) \end{bmatrix} + \begin{bmatrix} \underline{\beta}_1 \\ \underline{\beta}_2 \end{bmatrix} \underline{u}(k) + \begin{bmatrix} \underline{\Delta}_1 \\ \underline{\Delta}_2 \end{bmatrix} \underline{d}(k) \qquad 4.35$$

The relation governing the response of $\underline{z}_1$ is given by

$$\underline{z}_1(k+1) = \underline{\Phi}_1 \, \underline{z}_1(k) + \underline{\Phi}_2 \, \underline{z}_2(k) + \underline{\beta}_1 \, \underline{u}(k) + \underline{\Delta}_1 \, \underline{d}(k) \qquad 4.36$$

The first step in finding the control law is to divide $\underline{\Phi}_1$, into the diagonal matrix $\underline{\Phi}_d$ and the matrix $\underline{\Phi}_p$ with the original elements except for the main diagonal which is set to zero, such that

$$\underline{\Phi}_p + \underline{\Phi}_d = \underline{\Phi}_1 \qquad 4.37$$

Substituting into equation 4.36 gives

$$\underline{z}_1(k+1) = (\underline{\Phi}_p + \underline{\Phi}_d) \, \underline{z}_1(k) + \underline{\Phi}_2 \, \underline{z}_2(k) + \underline{\beta}_1 \, \underline{u}(k) + \underline{\Delta}_1 \, \underline{d}(k) \qquad 4.38$$

which upon rearrangement yields

$$\underline{z}_1(k+1) - \underline{\Phi}_d \, \underline{z}_1(k) = \underline{\Phi}_p \, \underline{z}_1(k) + \underline{\Phi}_2 \, \underline{z}\text{[illegible]}) + \underline{\beta}_1 \, \underline{u}(k) + \underline{\Delta}_1 \, \underline{d}(k) \qquad 4.39$$

This equation has the form

$$z_i(k+1) - \phi_{d_{ii}} \, z_i(k) = f_i \, (\underline{z}_1'', \underline{z}_2(k), \underline{u}(k), \underline{d}(k)) \qquad 4.40$$

where z_i - ith element of $\underline{z}_1$

$\underline{z}_1''$ - vector $\underline{z}_1$ with element i deleted

For invariance at the sampling instances in the control variable z_i it is necessary that

$z_i(k) = z_i(k+1) = z(k+n) = 0$, for n = any positive integer.

This means that the left hand side of equation 4.40 is zero so equation 4.39 becomes:

$$\underline{0} = \underline{\underline{\phi}}_p \, \underline{z}_1(k) + \underline{\underline{\phi}}_2 \, \underline{z}_2(k) + \underline{\underline{\beta}}_1 \, \underline{u}(k) + \underline{\underline{\Delta}}_1 \, \underline{d}(k) \qquad 4.41$$

Dividing $\underline{\underline{\beta}}_1$ into the diagonal matrix $\underline{\underline{\beta}}_d$ and the matrix with zero on the main diagonal $\underline{\underline{\beta}}_p$ such that

$$\underline{\underline{\beta}}_p + \underline{\underline{\beta}}_d = \underline{\underline{\beta}}_1 \qquad 4.42$$

and substituting this into equation 4.41 and rearranging gives

$$\underline{u}(k) = -(\underline{\underline{\beta}}_d)^{-1} \, (\underline{\underline{\phi}}_p \, \underline{z}_1(k) + \underline{\underline{\phi}}_2 \, \underline{z}_2(k) + \underline{\underline{\beta}}_p \, \underline{u}(k) + \underline{\underline{\Delta}}_1 \, \underline{d}(k)) \qquad 4.43$$

The control matrices can be given as

$$\hat{\underline{\underline{K}}}_{FF} = -\underline{\underline{\beta}}_d^{-1} \, \underline{\underline{\Delta}}_1$$

$$\hat{\underline{\underline{K}}}_{UC} = -\underline{\underline{\beta}}_d^{-1} \, \underline{\underline{\beta}}_p$$

$$\hat{\underline{\underline{K}}}_{DC} = -\underline{\underline{\beta}}_d^{-1} \, [\underline{\underline{\phi}}_p \underline{\underline{\phi}}_2][\underline{\underline{H}}_1 \; \underline{\underline{H}}_2]^{-1}$$

where $\hat{\underline{\underline{K}}}_{FF}$, $\hat{\underline{\underline{K}}}_{UC}$, $\hat{\underline{\underline{K}}}_{DC}$ - discrete control matrices for feedforward, input and output decoupling respectively.

This is the control function for the discrete case and the signal flow graph for this is given in Figure 4.4 with output feedback where

$$\underline{y}(k) = [\underline{\underline{H}}_1\ \underline{\underline{H}}_2] \begin{bmatrix} \underline{z}_1(k) \\ \underline{z}_2(k) \end{bmatrix} \qquad 4.44$$

so

$$\begin{bmatrix} \underline{z}_1(k) \\ \underline{z}_2(k) \end{bmatrix} = [\underline{\underline{H}}_1\ \underline{\underline{H}}_2]^{-1}\ \underline{y}(k) \qquad 4.45$$

where dim $\underline{y}(k)$ = dim $(\underline{z}(k))$

As in the continuous case the analytic solution of equation 4.41 gives an approximate solution lacking explicit control input decoupling. This is given as

$$\underline{u}(k) = -\underline{\underline{\beta}}_1^{-1} (\underline{\underline{\phi}}_p\ \underline{z}_1(k) + \underline{\underline{\phi}}_2\ \underline{z}_2(k) + \underline{\underline{\Delta}}_1\ \underline{d}(k)) \qquad 4.46$$

The motivation for using this form is greater for the discrete case as the algebraic loop including $\hat{K}_{UC}$ must be solved by an iteritive method at each sample interval. The addition of feedback control such as the discrete approximation of P + I control given by

$$W_i(k) = K_p\ z_i(k) + K_I \sum_{j=0}^{k} (z_i(j)\ T) \qquad 4.47$$

For the case with explicit decoupling

$$\underline{u}(k) = \underline{\underline{\beta}}_d^{-1} [\underline{\underline{\phi}}_p\ \underline{z}_1(k) + \underline{\underline{\phi}}_2\ \underline{z}_2(k) + \underline{\underline{\beta}}_p\ \underline{u}(k) + \underline{\underline{\Delta}}_1\ \underline{d}(k)] + \underline{W}(k) \qquad 4.48$$

and for the simplified control as

$$\underline{u}(k) = \underline{\underline{\beta}}_1^{-1} (\underline{\underline{\phi}}_p\ \underline{z}_1(k) + \underline{\underline{\phi}}_2\ \underline{z}_2(k) + \underline{\underline{\Delta}}_1\ \underline{d}) + \underline{W}(k) \qquad 4.49$$

This is illustrated with a signal flow graph in Figure 4.5.

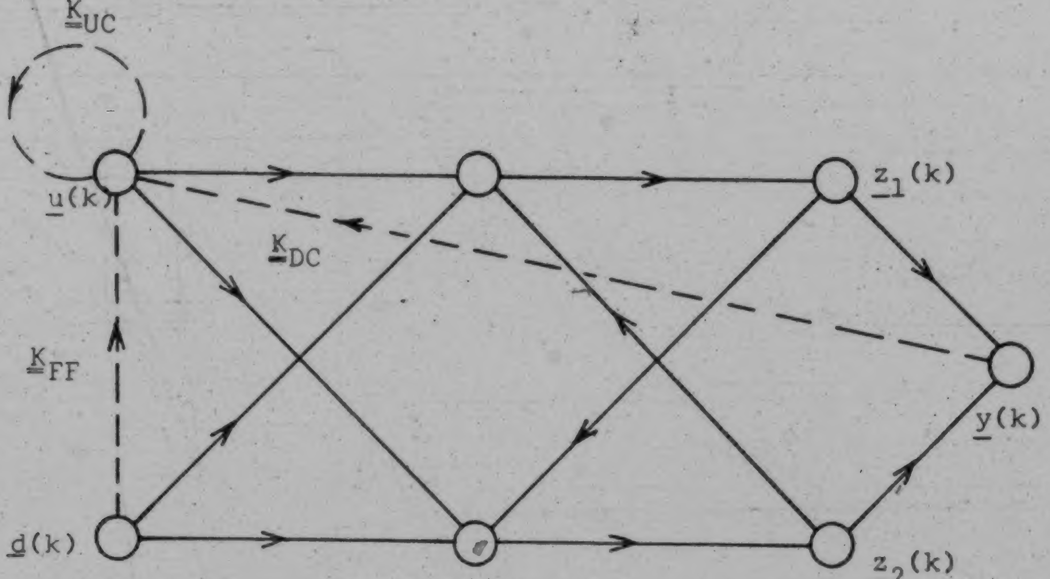


FIGURE 4.4: Signal Flow Graph of Discrete System with Feedforward and Decoupling Control.

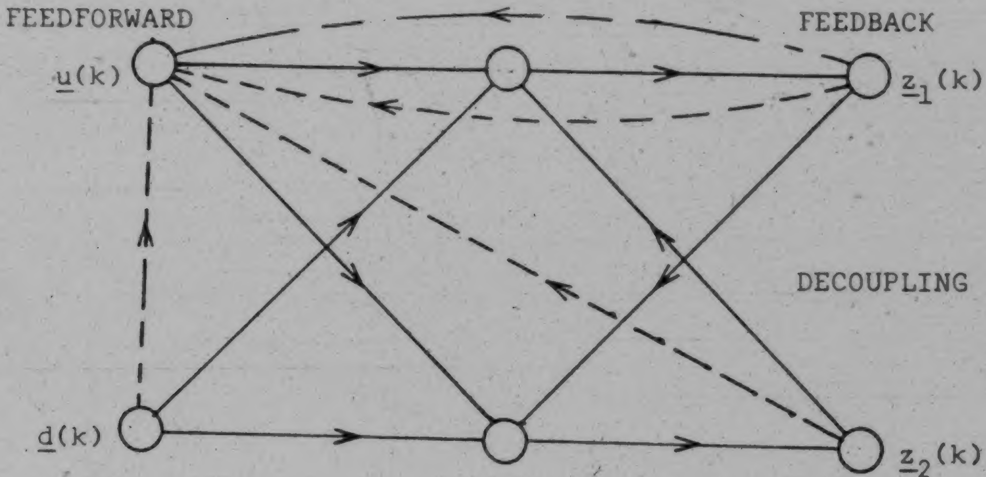


FIGURE 4.5: Signal Flow Graph of Discrete System with Simplified Feedforward and Decoupling Scheme with Feedback Loop Shown.

4.3 Implementation of the Control Scheme

The method of deriving feedforward and decoupling control for multivariable systems was applied to the pilot scale distillation column at the University of Alberta. The design procedure used to implement control included first the calculation of the control law from the model of the system then a simulation study followed by verification of the effectiveness of the scheme on the experimental equipment.

4.3.1 Simulation

The model derived in the previous chapter was used to calculate the control laws. The linear continuous version of the distillation column model was used with equation 4.33 to obtain the corresponding control matrices. It was found that the control matrices were singular when the control variables were the terminal compositions or the first and tenth states. This was due to a row of zeros in $\underline{\underline{B}}_1$ in equation 4.33 corresponding to the lack of direct effect of the inputs on the overhead composition. From the model matrices given in Appendix B it can be seen that the only variable that affects the overhead composition is the composition on the adjacent stage. It is therefore logical to use this as the control variable in place of the overhead composition.

The control matrices resulting from the calculation using the bottoms composition and the composition of the liquid on stage nine (eighth tray) are given in Appendix D.

Due to the tridiagonal form of the model of the system it is necessary to measure only states two, eight and ten in order to decouple the system.

The response of the system with the calculated control to a ten percent increase in feed was predicted using CSMP III (56). This is an IBM program developed for solving differential equations. The predicted response using the linear tenth order continuous model gave almost no deviation from steady state as can be seen from the final deviation given in Table 4.1 run 4.1.1. The control function used here does not include feedback but from the results they do not appear to be required. In order to give a more realistic prediction the control matrices were used with the non-linear model of the system. Even though the results were not as good as the previous ones they still indicated that almost invariant control of the terminal compositions can be expected as seen by the final deviation given in Table 4.1 run 4.1.2.

Since the actual control function for the physical system is carried out at discrete one minute intervals by a program on the IBM 1800 digital computer the discrete control law was found. In order to do this it is necessary to discretize the system model. The discrete model was calculated using a program in GEMSCOPE. GEMSCOPE is a collection of programs written in the Department of Chemical Engineering at the University of Alberta to carry out control problem calculations. These are described by Wilson (51) who linked

TABLE 4.1

DEVIATION OF TERMINAL COMPOSITIONS PREDICTED FROM TENTH ORDER LINEAR CONCENTRATION MODEL WITH FEEDFORWARD DECOUPLING FUNCTION FOR A TEN PERCENT STEP INCREASE IN FEED FLOW

Run	Order Model Used for Control Calculation	Deviation of Terminal Compositions from Initial Steady State (Wt. Pct. MEOH)	
		Bottoms Composition	Overhead Composition
4.1.1	10	5.6×10^{-7}	-4.0×10^{-6}
4.1.2	10	9.6×10^{-4}	$\sim 10^{-5}$
4.1.3	10	5.3×10^{-8}	-4.9×10^{-8}
4.1.4	4	-5.4×10^{-7}	4.0×10^{-6}
4.1.5	4*	-9.0×10^{-7}	5.7×10^{-7}
4.1.6	2	8.7×10^{-6}	2.0×10^{-7}

* Removed element corresponding to feedback from X_D to D

them into a convenient form for the IBM 360 operating under the MTS system. The resulting tenth order model matrices of the discrete linear model with a one minute sample time are given in Appendix D. The response of this model was compared to that of the continuous version for a ten percent increase in feed and found to agree well. The control matrices were calculated from the discrete model according to equation 4.46 and the response with the control to a ten percent increase in feed predicted using a program in GEMSCOPE. The control matrices are given in Appendix D and the final steady state of the control run in Table 4.1, run 4.1.3. The discrete model as used to calculate the controlled response resulted in almost invariant response and so this form of control would seem best. However, in discretizing the model the tridiagonal form of the model matrix was lost and so the decoupling portion required that all the states be measured. This was thought wasteful of computer time and storage space.

In order to determine if simplified control was feasible it was decided to reduce the model. The GEMSCOPE program contains two different model reduction routines, modal reduction proposed by Marshall (26) and least squares reduction proposed by Anderson (1) and modified by Wilson (52). The tenth order model was first reduced to a fourth order model retaining the terminal compositions and the liquid compositions of the second and ninth stages. The results from both reduced models (one from least squares and one by modal reduction) gave excellent final steady state agreement with the predicted final steady state of the tenth order discrete model as can be seen in Table 4.2, runs 4.2.2, 4.2.3 and 4.2.4. However, the dynamic agreement of the model reduced by the modal technique was by far the better of the two as can be seen in Figures 4.6 and 4.7. Reduction to second order gave results similar to those of the fourth order models as shown in Table 4.2 and Figure 4.7.

The control functions were calculated using the fourth and second order reduced models using equation 4.46 and the resulting control matrices are presented in Appendix D.

The predicted controlled response to a ten percent increase in feed from the tenth order model was calculated using the fourth and second order control matrices. As can be seen from the final steady states given in Table 4.1, runs 4.1.4 and 4.1.5, the terminal compositions did not deviate significantly indicating that essentially invariant control was achieved.

It was noted that the feedback decoupling matrix had a negative element corresponding to feedback from the overhead composition to the overhead flowrate in the fourth order control law. For single loop control this would result in unstable operation. A simulation run was carried out with this element set to zero. This resulted in almost no change in the predicted final steady state as can be seen in Table 4.1, run 4.1.5.

TABLE 4.2

PREDICTED OPEN LOOP DEVIATION OF TERMINAL COMPOSITIONS TO A TEN PERCENT INCREASE IN FEED FLOW USING REDUCED MODELS

Run	Order of Model	Reduction Technique	Deviation of Terminal Compositions from Initial Steady State (Wt. Pct. MEOH)	
			Bottoms Composition	Overhead Composition
4.2.1	10		.03524	.0001704
4.2.2	4	Modal	.03525	.0001705
4.2.3	4	Least Squares	.03526	.0001707
4.2.4	2	Modal	.03526	.0001706
4.2.5	2	Least Squares	.03526	.0001707

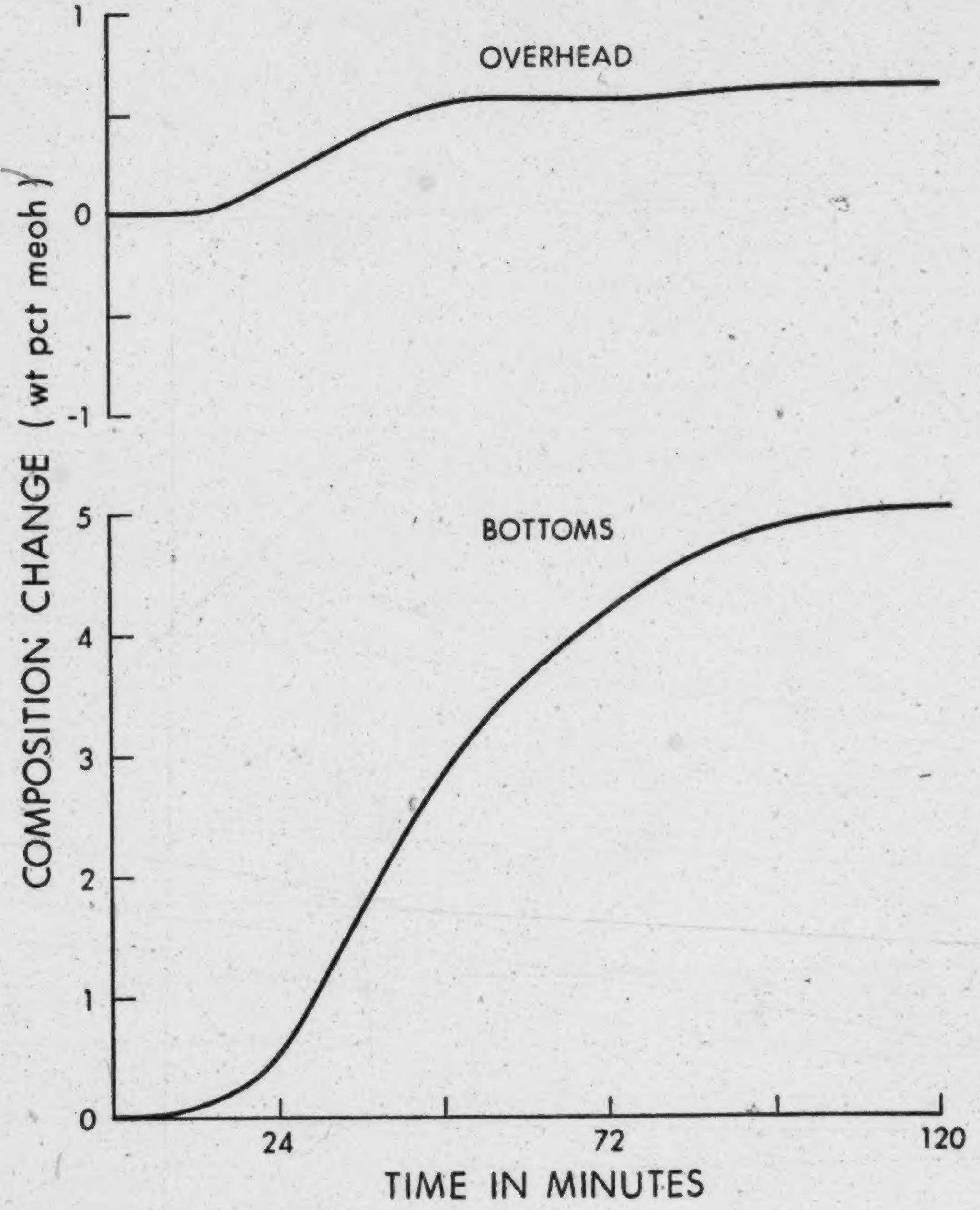


FIGURE 4.6: Experimental Open Loop Response to a Ten Percent Step in Feed (2.4-2.64 LB/MIN)

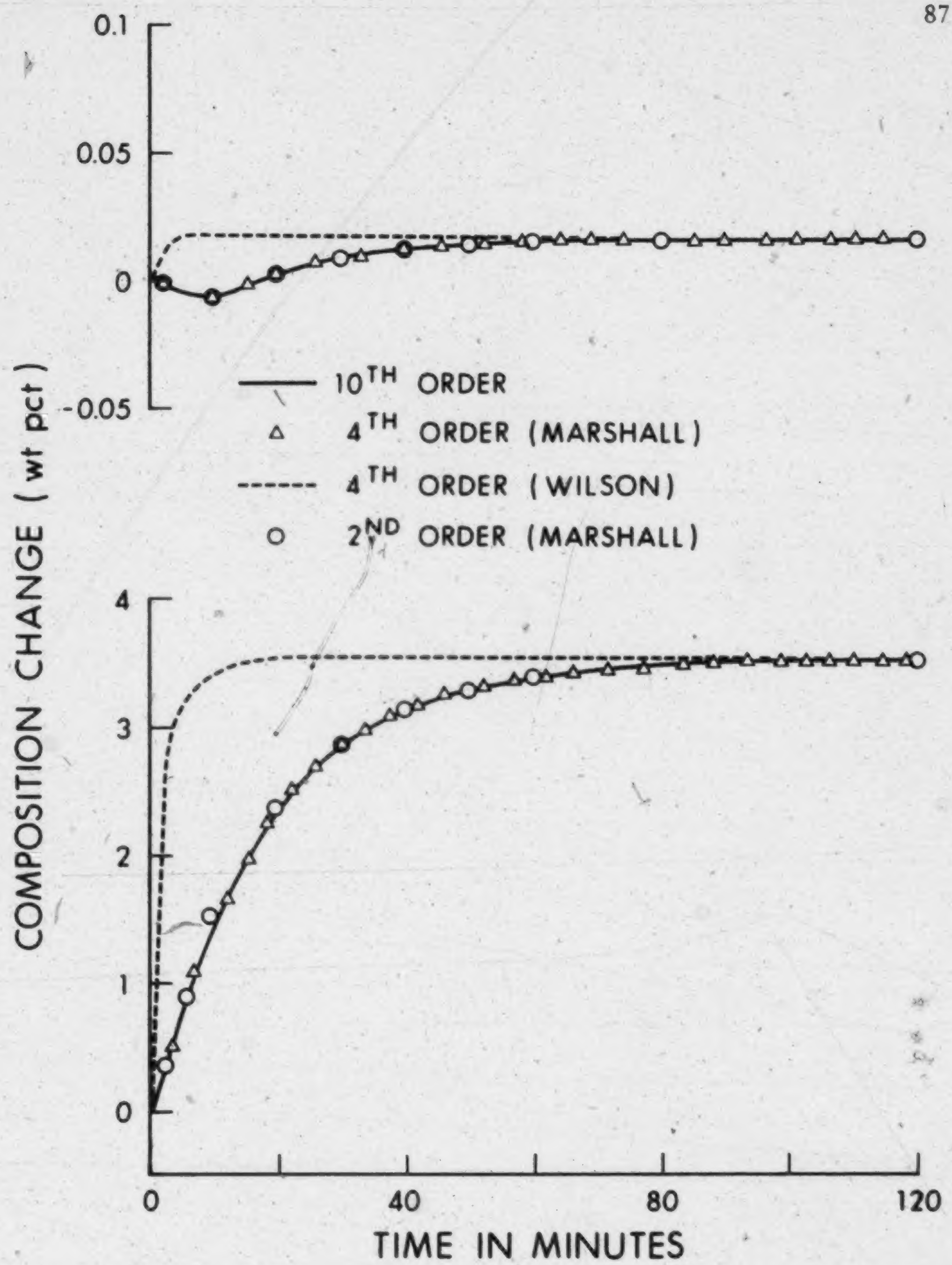


FIGURE 4.7: Comparison of Predicted Response of Tenth Order Linear Discrete Model and Second and Fourth Order Reduced Model for a Ten Percent Step in Feed

Consideration of the results from the discrete simulation reveals that the predicted responses of the tenth order model with the various control matrices exhibited final offsets which are far smaller than can be measured experimentally. Thus it does not appear to matter which set of control matrices is chosen from a control quality standpoint. However, two other features must be considered in implementing the scheme on the actual system. First, measurement noise and unreliability present in the physical system can render a sensitive system such as one with large feedback elements inoperative. Secondly, the model of the process is not perfect, for example, the distillation column is quite non-linear while the control law formulation is based on a linear system representation which means offset can be expected on any disturbance.

In the simulation runs the addition of the feedback loops was not needed because the feed disturbance introduced could be adequately attenuated by the feedforward action of the control law. However, as feedback will probably be required in the experimental study some simulation was carried out with integral feedback loops to determine what the effect will be. In order to study the effect of feedback the feedforward control was removed and integral feedback loops were added to the overhead and bottoms compositions. The results of the tests made are shown in Table 4.3. Runs 4.3.1 and 4.3.2 show the effect of removing the feedforward action leaving only the decoupling function. The

TABLE 4.3

PREDICTED DEVIATIONS OF TERMINAL COMPOSITIONS FOR A TEN PERCENT INCREASE IN FEED FLOW USING THE TENTH ORDER CONCENTRATION MODEL WITH DECOUPLED FEEDBACK CONTROL

	Deviation of Composition at 100 Min (Wt. Pct. MEOH)			
Run	Bottoms	K_I	Overhead	K_I
4.3.1*	.0029	0	-.00303	0
4.3.2	.0032	0	-.00056	0
4.3.3	.0023	100.	.0002	1.
4.3.4	.0033	1.	$.5 \times 10^{-5}$	1.
4.3.5	.0034	0	$.3 \times 10^{-5}$	1.
4.3.6	.000067	1000.	$.9 \times 10^{-4}$	1.
4.3.7	$.25 \times 10^{-5}$	10000.	$.2 \times 10^{-4}$	1.

* Negative element in feedback decoupling matrix retained.

resulting final steady state is different from the open loop run for which the final steady state is given in Table 4.2, run 4.2.1. Removal of the negative element in the decoupling matrix corresponding to feedback from the distillate composition to the distillate flowrate decreases the final steady state offset of the distillate composition, as can be seen from Table 4.3, and furthermore, the overall control performance is improved. It is interesting to note that both increasing and decreasing the bottoms integral constant from 100 resulted in a decrease in the final overhead composition offset. The bottoms composition deviation at final steady state decreases up to an integral constant of ten thousand at which point some evidence of oscillation was present but the system remained stable.

The noise level in the measurement devices is an important feature of the experimental equipment. The gas chromatograph used to measure bottoms composition has a very high noise level as can be seen from the steady state data plotted in Appendix A. There is a time delay of three minutes for the analysis in addition to the delay resulting from exponential filtering. Although the tray temperatures which are used to estimate liquid compositions also have relatively high noise levels, they give better results than the gas chromatograph.

The control law derived from the fourth order discrete model was chosen for the experimental study. It has the advantage of allowing some internal state measurements to supplement the terminal composition measurements while still having small dimension thus saving computer time and storage.

The success of the simulation study still leaves unanswered the question of performance of the continuous control function with the experimental equipment and also what will be the effect of system noise upon performance. Consequently, the experimental study was undertaken to attempt to provide answers to these questions.

4.3.2 Experimental

The object of the control study was to obtain a simple efficient control scheme for the pilot plant distillation column.

The first phase of the experimental study involved testing the continuous control law. The system was configurated as shown in Figure 4.8. On initiation of control at steady state monitoring the terminal compositions revealed oscillations of increasing magnitude to the system limits before the feed disturbance was introduced. The system was unstable so it was necessary to put the control law in the discrete form.

Control of the column was then attempted using the simplest discrete control law resulting from the use of the second order model representation. It was found that the system oscillated due to the high noise level in the gas chromatograph measurement of bottoms composition. The

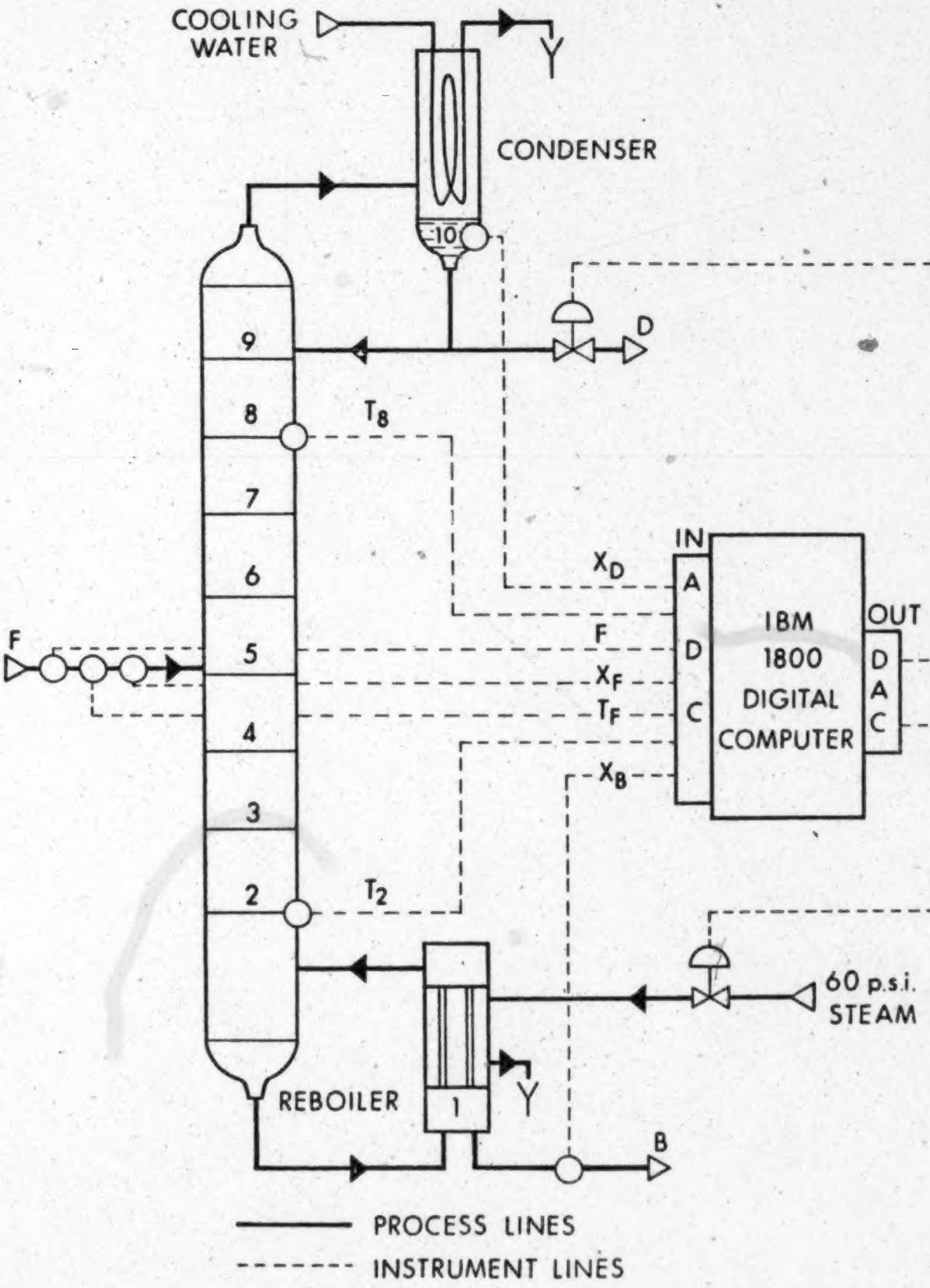


FIGURE 4.8: Schematic of Column Control Configuration for Continuous Control Law

system was stable but control was found to be unsatisfactory.

The fourth order model which was considered the logical choise was used to find the control law, the numerical values for which are given in Appendix D. A simplified schematic of the system layout is shown in Figure 4.9. The final steady state deviations of the controlled variables are given in Table 4.4 and the corresponding feedback constants and filter constants used for the bottoms composition measurement are given in Table 4.5.

The response of the system with the fourth order feedforward control function to an increase and decrease of ten percent are shown in Figures 4.10 and 4.11 respectively. The disturbance is well attenuated but a final offset occurs in both runs.

The addition of feedback control with decoupling action resulted in the improved response to feed flow changes as shown in Figures 4.12 and 4.13. The deviation in both compositions appears to be less than the noise level and so control was considered to be satisfactory.

Figures 4.14 and 4.15 show the effect of increasing the integral feedback constants, using the values given in Table 4.5, where it appears that the increase in integral action gives some improvement in the control. To test this conclusion, tests were made using a series of random disturbances. The results as shown in Figure 4.16 and 4.17 reveal that the higher integral constants tend to make the compositions oscillate and for these larger disturbances

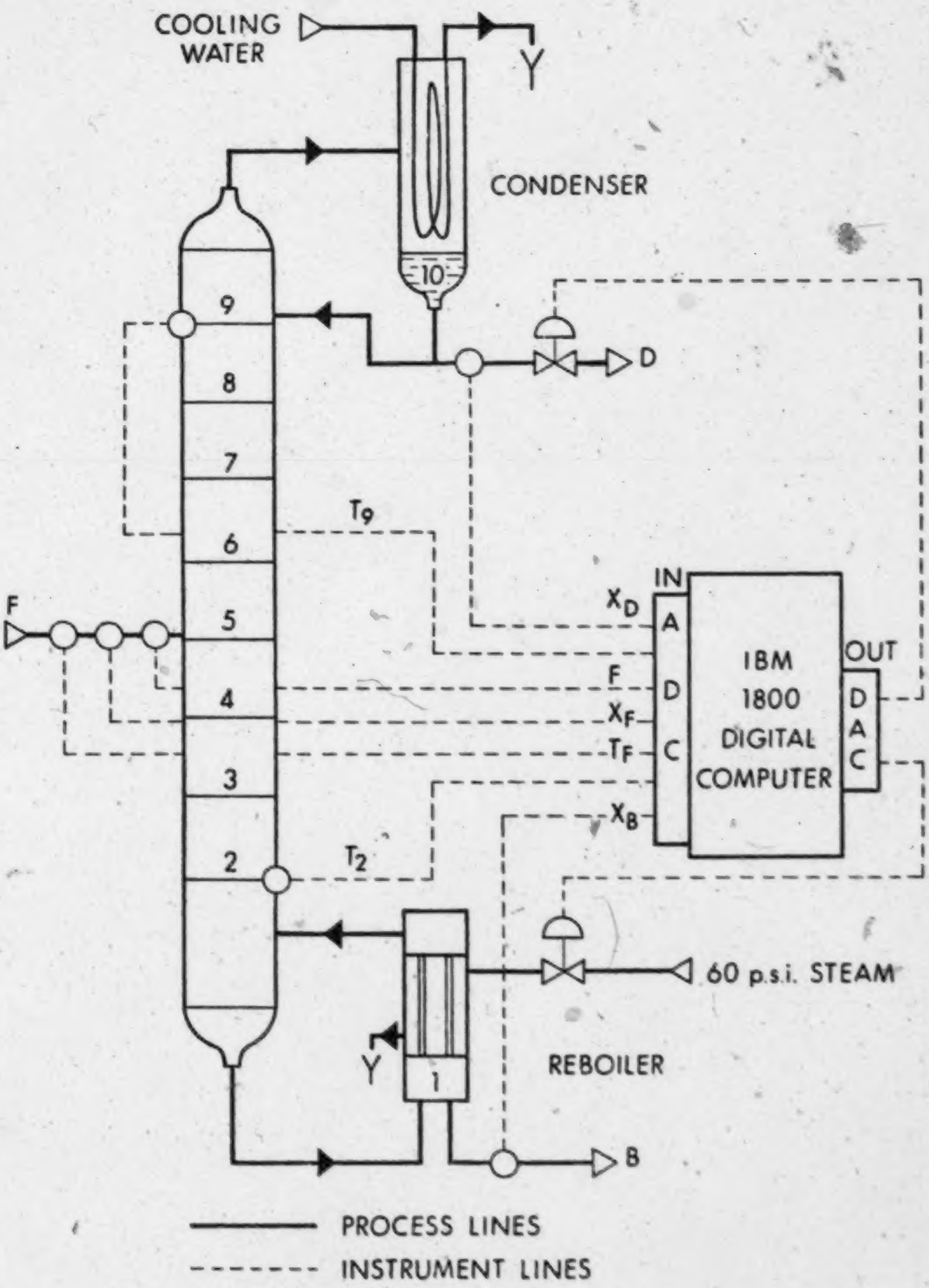


FIGURE 4.9: Simplified Schematic of Column Control for Discrete Control Law

TABLE 4.4

SUMMARY OF EXPERIMENTAL RESULTS FOR CONTROL TESTS

Run	Input	Control Action	Bottoms Composition (Wt. Pct. MEOH)				Overhead Composition (Wt. Pct. MEOH)				Figure
			Steady State		IAE	IE	Steady State		IAE	IE	
			Initial	Final			Initial	Final			
1	10% F	FF+DC	1.2	1.2	88.9	31.8	97.4	97.4	41.2	40.5	
2	-10% F	FF+DC	4.7	3.0	53.5	46.4	98.3	99.1	43.2	43.1	
4	10% F	FF+DC	1.6	1.6	21.4	14.5	97.4	98.4	42.3	39.8	
5	10% F	FF	1.8	2.4	39.7	13.6	96.7	97.3	40.8	40.3	4.10
6A	-10% F	FF	3.9	3.3	61.1	46.0	98.7	98.3	19.8	19.5	4.11
7	10% F	FF+DC+FB	2.8	2.7	27.0	18.2	98.1	98.0	8.8	1.4	4.12
8	-10% F	FF+DC+FB	2.0	1.8	31.3	.2	97.7	97.6	9.2	7.9	4.13
9	20% F	FF+DC+FB	1.6	1.9	38.4	29.6	97.5	97.3	15.2	11.3	4.18
11	$-T_F$	FF+DC+FB	2.0	2.0			98.3	98.4			4.21
13	15% F	FF+DC+FB	2.0	2.2			97.3	97.1			
14	10% F	FF+DC+FB	2.0	2.0	25.1	1.6	97.0	97.0	6.6	1.3	4.14
16	-10% F	FF+DC+FB	1.8	1.8	17.3	4.8	97.2	97.2	9.2	2.2	4.15
20	RI	FF+DC+FB									4.16
21	RI	FF+DC+FB									4.17

TABLE 4.4 (Continued)

Code Designation for Input and Control Action:

Input

10% F - Change in Feed Flow 2.4-2.64 lb/min
-10% F - Change in Feed Flow 2.4-2.16 lb/min
20% F - Change in Feed Flow 2.4-2.88 lb/min
15% F - Change in Feed Flow 2.4-2.76 lb/min
$-T_F$ - Change in Feed Temp. 161-141°F

Control Action

FB - Feedback Control
DC - Elements to Decouple Feedback Control
FF - Decoupled Feedforward Control

Other

IAE - Time Integral of the Absolute Value of the Error
IE - Time Integral of the Error
RI - Random Input

TABLE 4.5

FILTERING AND FEEDBACK CONTROLLER CONSTANTS USED IN EXPERIMENTAL TESTS

	Exponential Filter Constant +		Integral Controller Feedback Constants (sec)	
Run	β^*_{Actual} (min)	β^*_{DDC}	Bottoms Loop	Overhead Loop
1	0	0	0	0
2	0	0	0	0
4	0.5	0.84315	0	0
5	0.5	0.84315	0	0
6A	0.5	0.84315	0	0
7	0.5	0.84315	100.	1.
8	0.5	0.84315	100.	1.
9	0.5	0.84315	100.	1.
11	0.5	0.84315	100.	1.
13	0.5	0.84315	100.	1.
14	0.5	0.84315	1000.	2.
16	.75	0.92625	1000.	2.
20	.75	0.92625	1000.	2.
21	.75	0.92625	100.	1.

+ Constant used in bottoms composition, x_B, filter equation $x_B(i)=x_B(i-1)(1-\beta) + \beta x(i)_{measured}$

* β_{Actual} - Constant based on one minute sample time

β_{DDC} - Constant based on actual sample time of 16 seconds

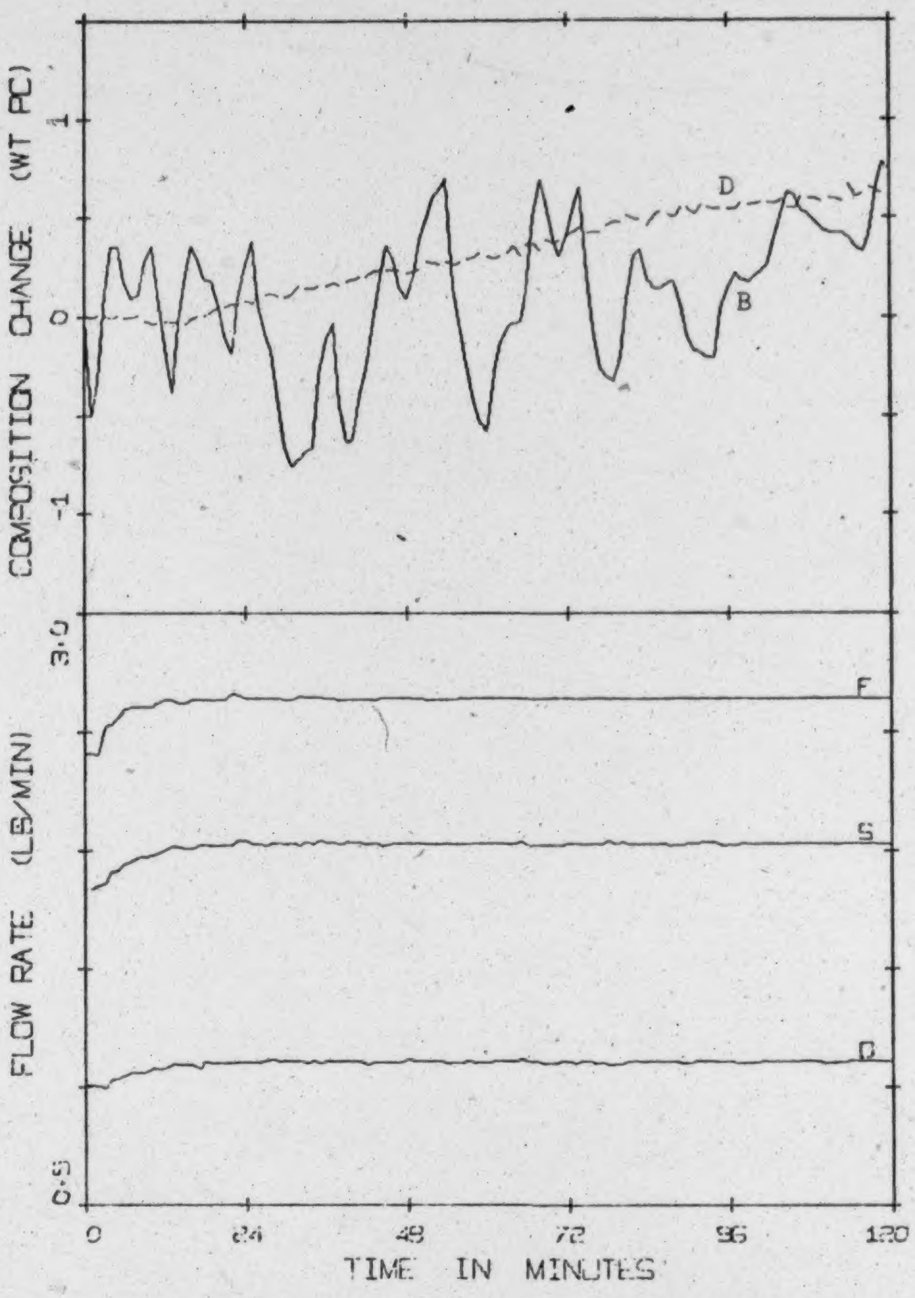


FIGURE 4.10: Experimental Response with Decoupled Feedforward Control for a 10% Step in Feed (2.4-2.64 LB/MIN).

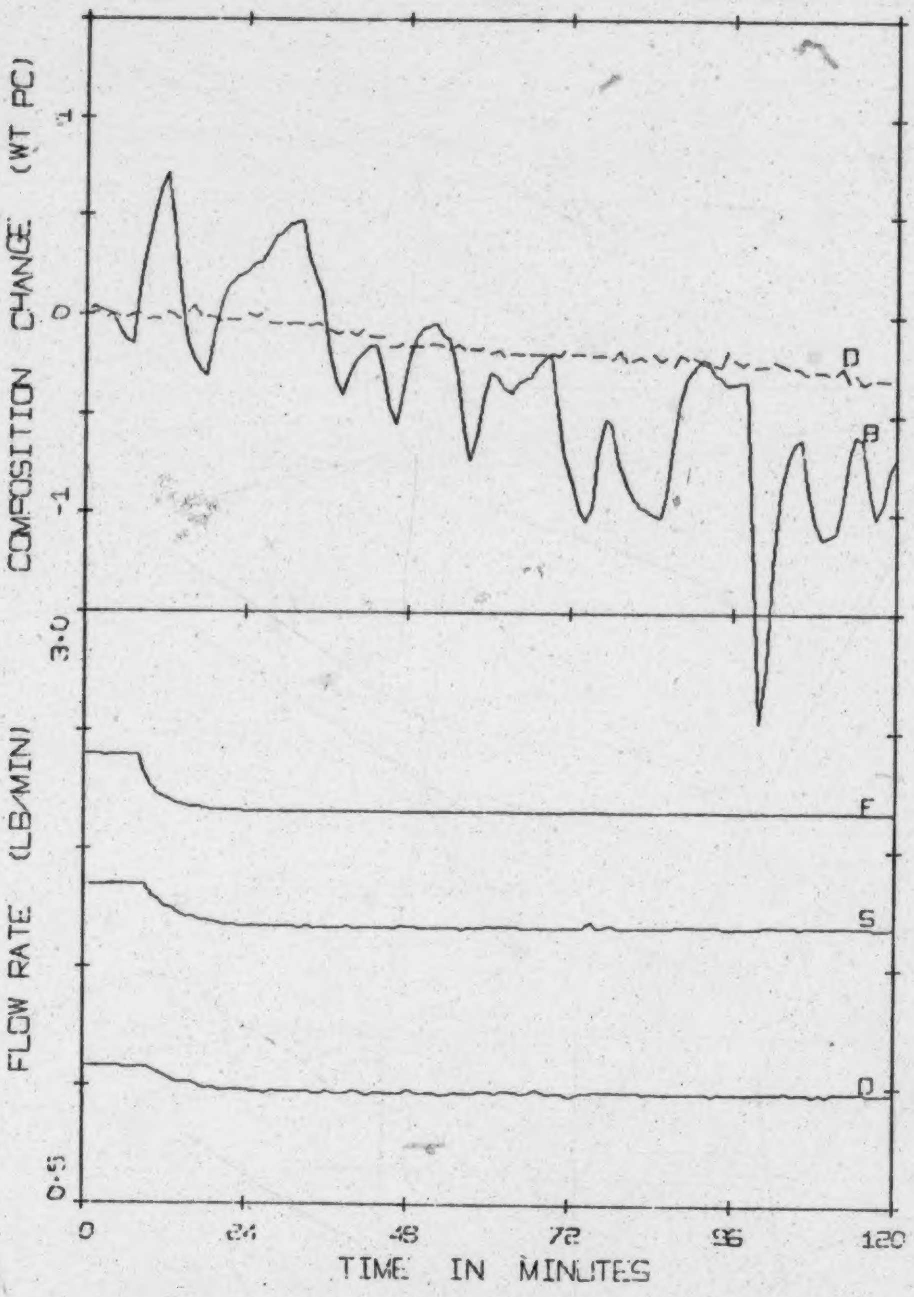


FIGURE 4.11: Experimental Response with Decoupled Feedforward Control for a -10% Step in Feed Flow (2.4-2.16 LB/MIN).

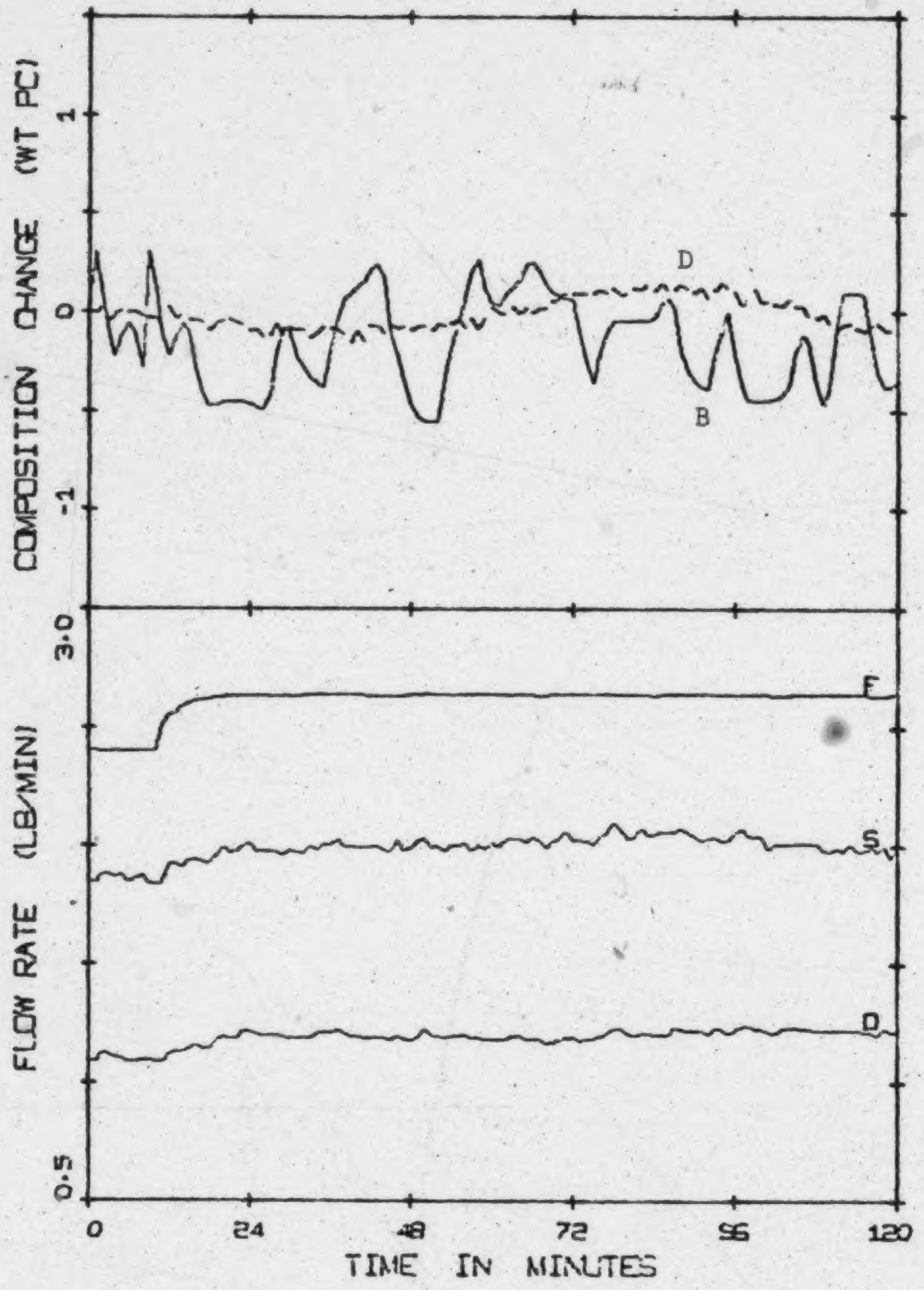


FIGURE 4.12: Experimental Response with Decoupled Feedforward and Feedback Control for a 10% Step in Feed (2.4-2.64) ($\underline{K}_I$=[1,100]).

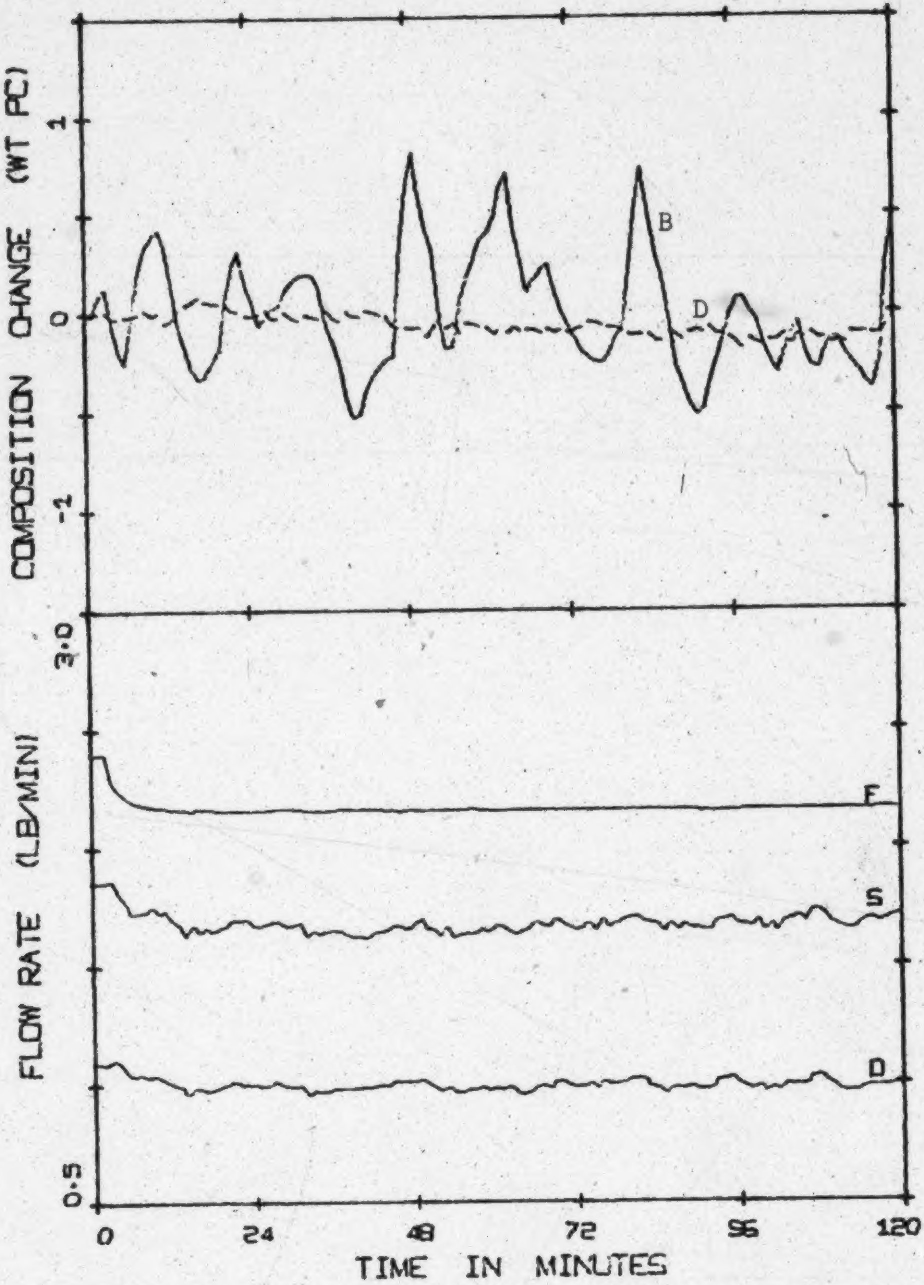


FIGURE 4.13: Experimental Response with Decoupled Feed-forward and Feedback Control for a -10% Step in Feed Flow (2.4-2.16 LB/MIN) ($\underline{K}_I$=[1,100]).

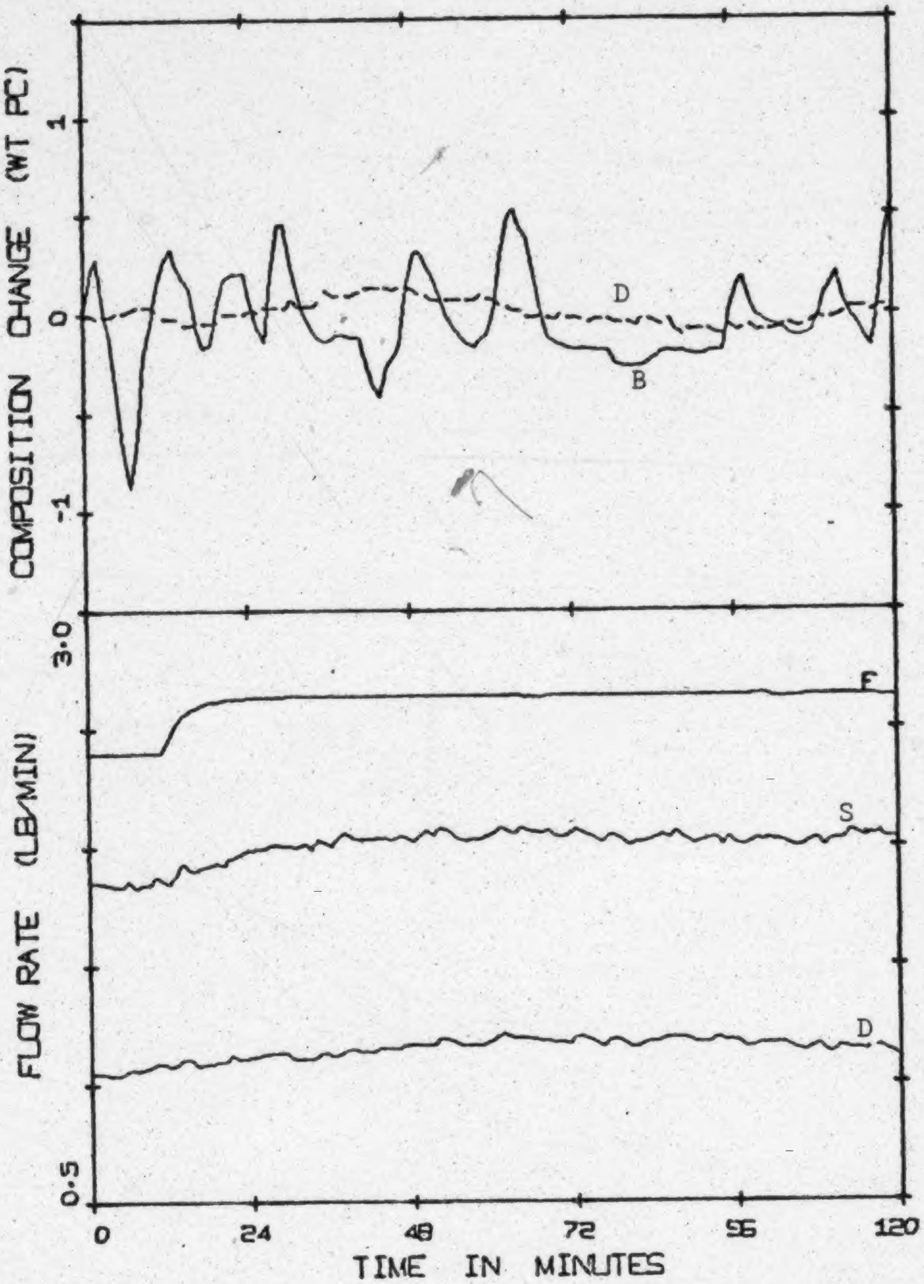


FIGURE 4.14: Experimental Response with Decoupled Feedforward and Feedback Control for a 10% Step in Feed Flow (2.4-2.64 LB/MIN) ($\underline{K}_I$=[2,1000]).

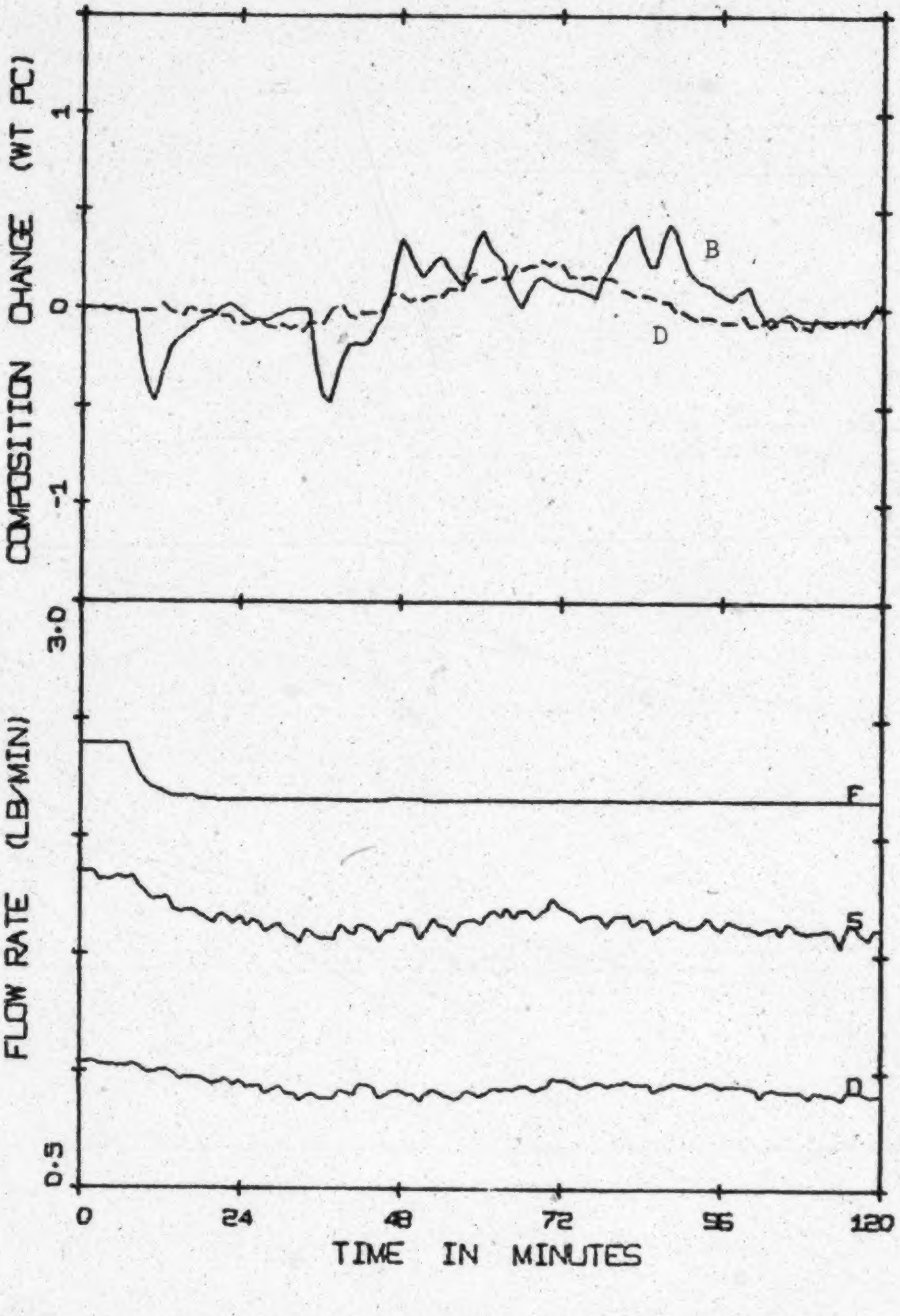


FIGURE 4.15: Experimental Response with Decoupled Feed-forward and Feedback Control for a -10% Step in Feed Flow (2.4-2.16 LB/MIN) ($\underline{K}_I$=[2,1000]).

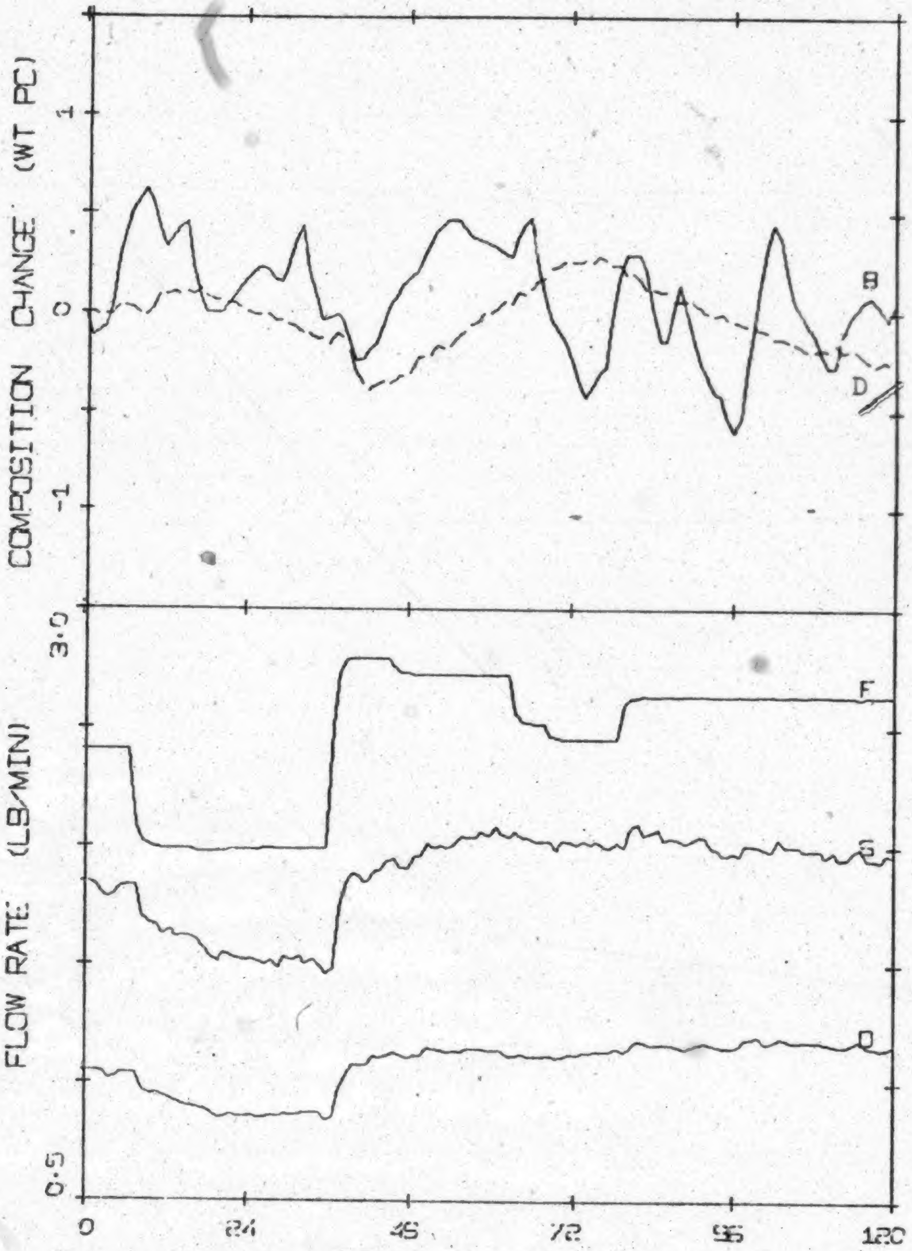


FIGURE 4.16: Experimental Response with Decoupled Feedforward and Feedback Control to a Series of Disturbances ($\underline{K}_I$=[1,100])

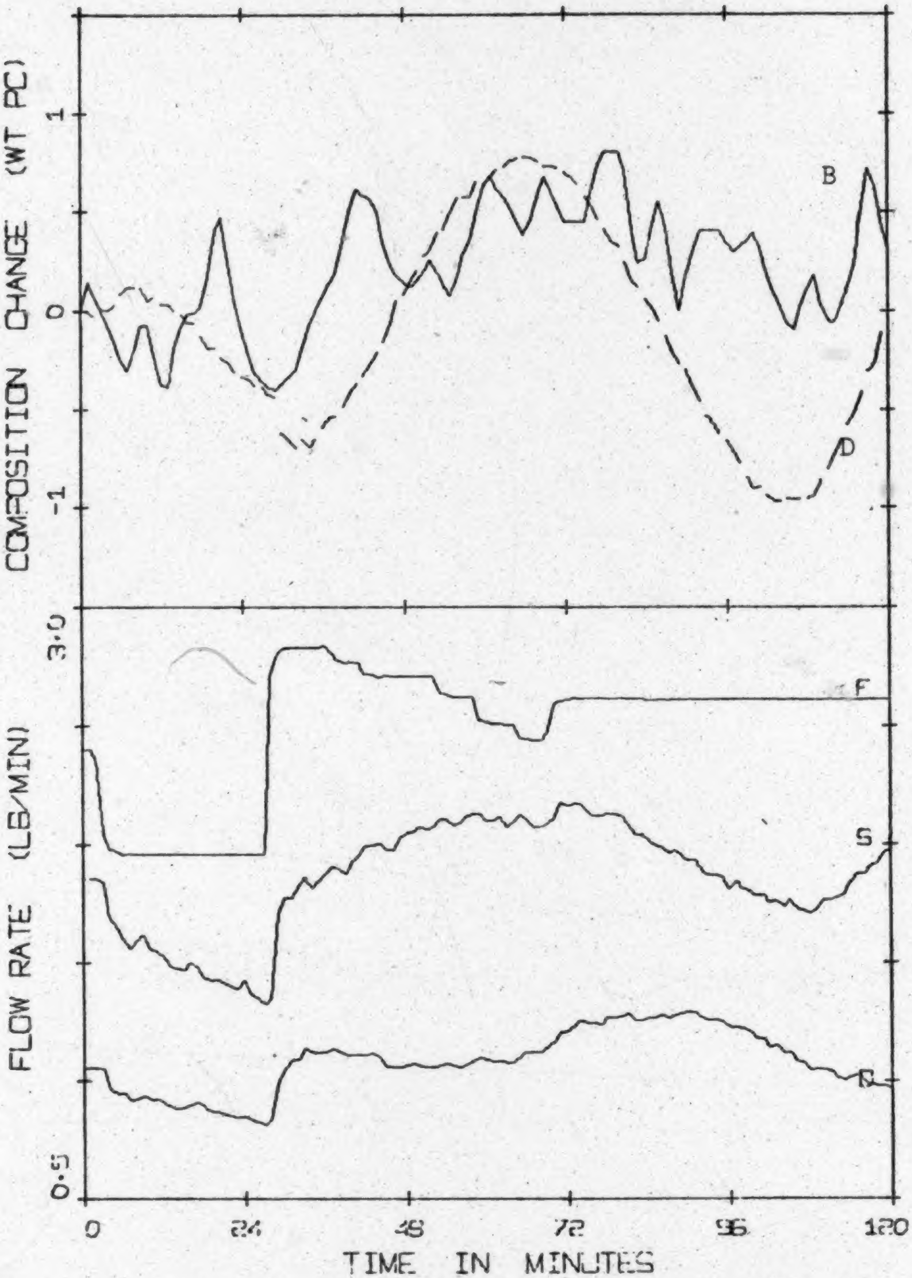


FIGURE 4.17: Experimental Response with Decoupled Feedforward and Feedback Control to a Series of Disturbances ($\underline{K}_I$=[2,1000])

they result in poorer control.

Figures 4.18 to 4.20 give some results from the work of Berry and McGinnis for comparison purposes. The run of Berry is comparable to results obtained in this study. McGinnis' control scheme showed oscillatory behavior due to the noise in the bottoms composition measurement. McGinnis used a second order discrete control law which was comparable to the second order scheme tried in this study but did some intuitive "tuning" to make it work.

Figure 4.21 shows the response of the control variables to a change in feed enthalpy. This test was carried out to ensure that other types of disturbances would be compensated for by the scheme employed in this study.

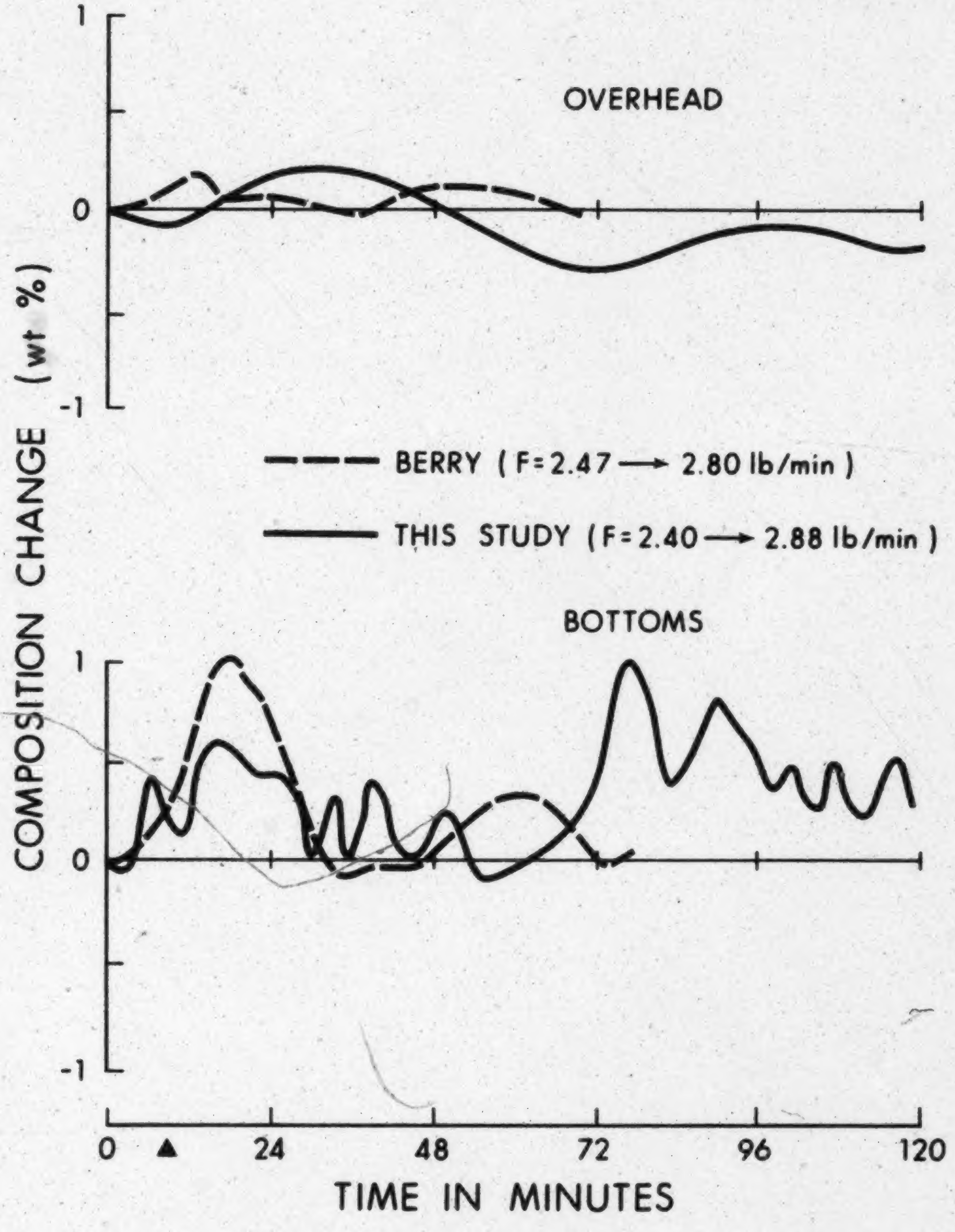


FIGURE 4.18: Experimental Response Obtained by Berry With Decoupled Feedback Control Compared to Response With Decoupled Feedforward Feedback Control for an Increase in Feed Flow.

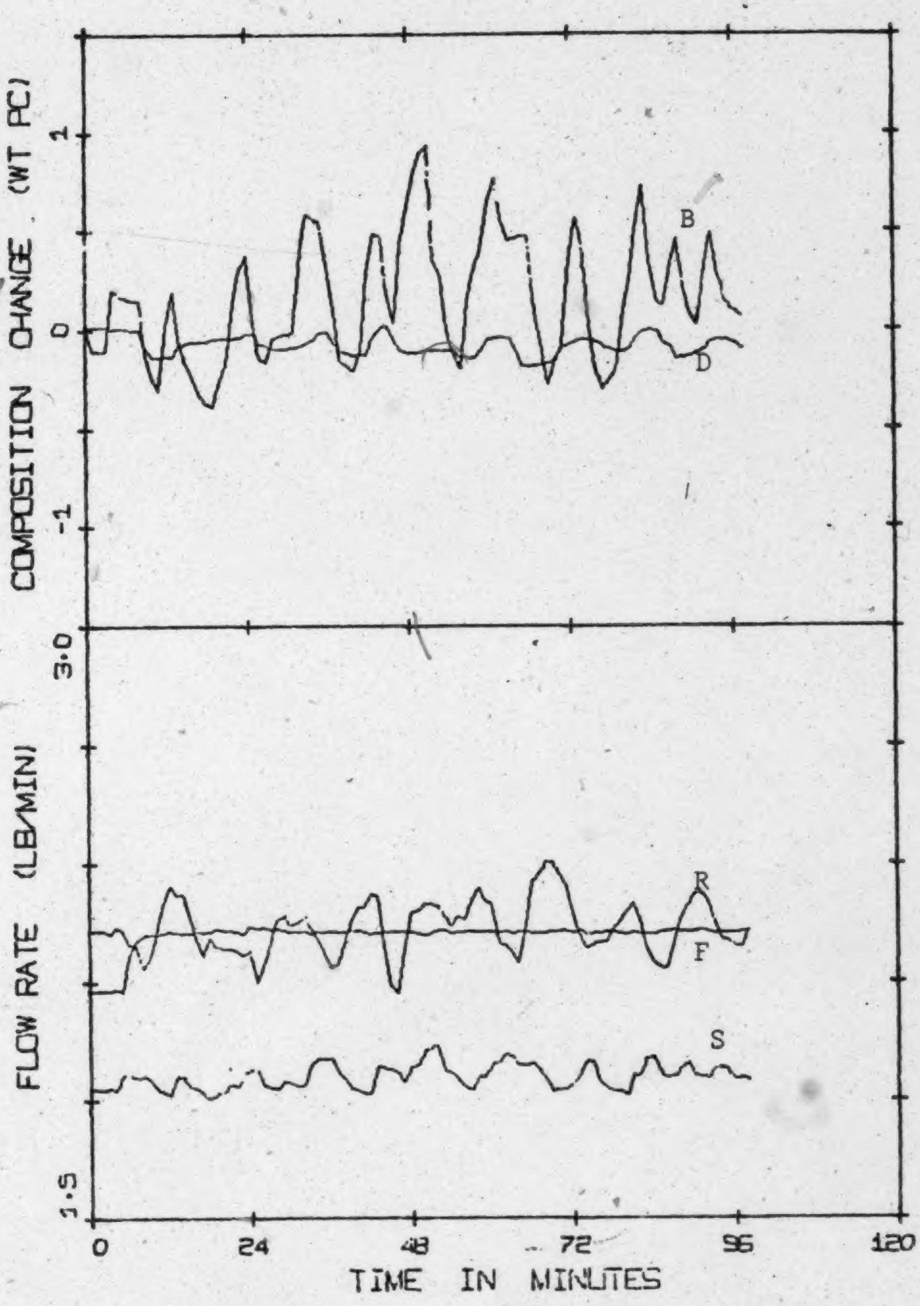


FIGURE 4.19: Experimental Response Obtained by M^{c}Ginnis with Multivariable Control for a 5% Change in Feed Flow.

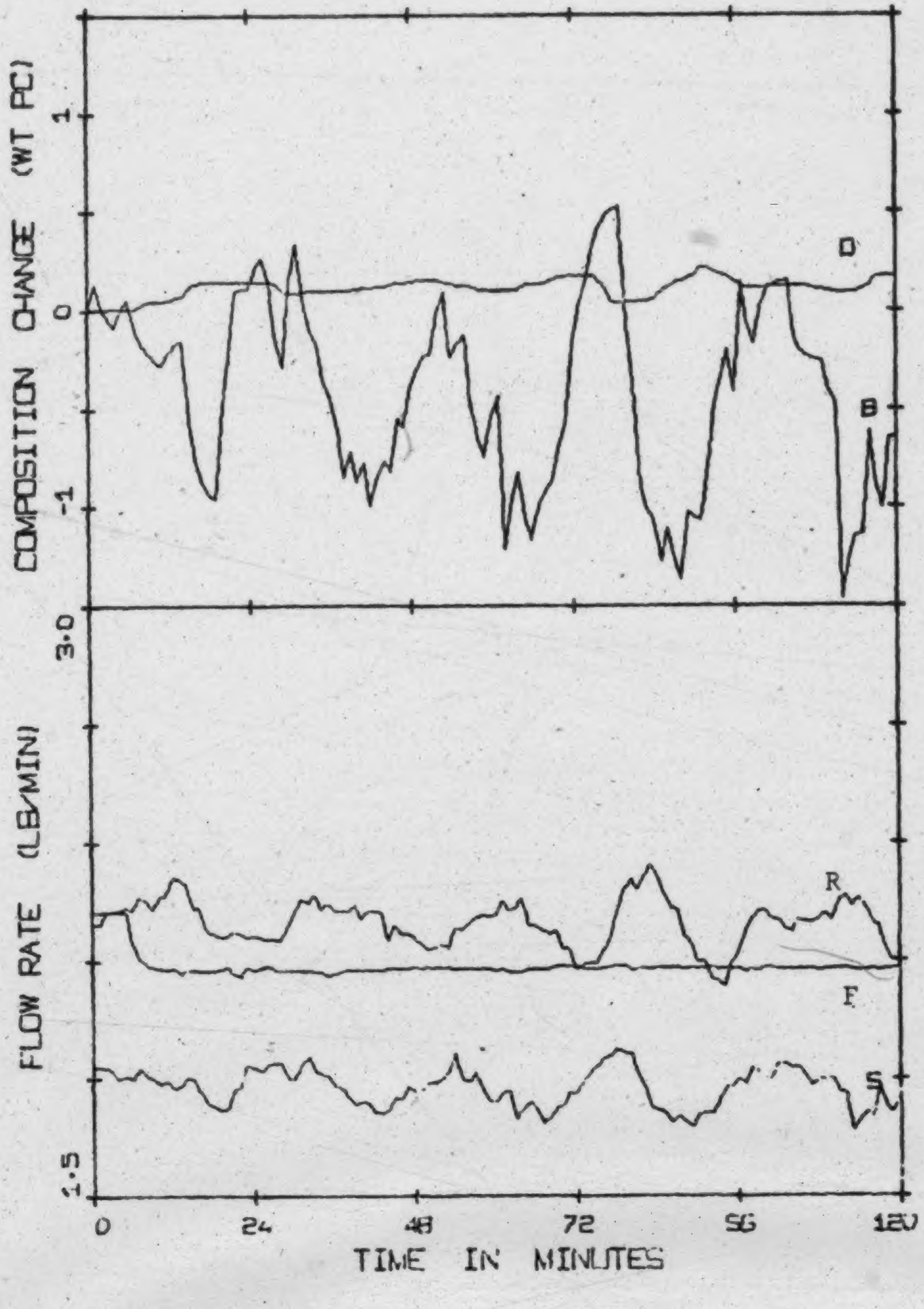


FIGURE 4.20: Experimental response Obtained by $M^cGinnis$ with Multivariable Control for a -5% Change in Feed Flow.

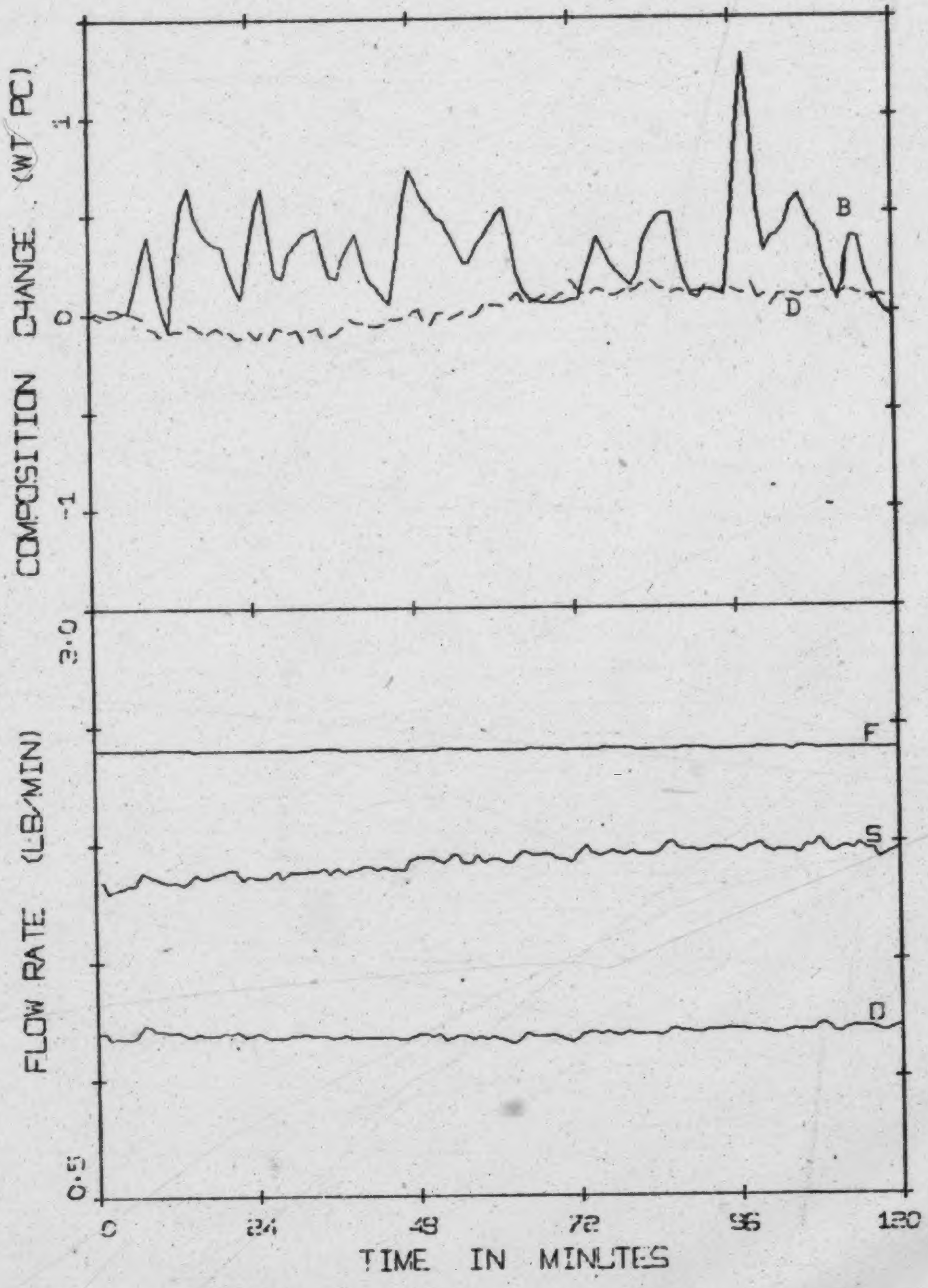


FIGURE 4.21: Experimental Response with Decoupled Feedforward and Feedback Control for a Change in Feed Temperature from 161-141°F ($\underline{K}_I$=[1,100]).

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions can be drawn from the three phases of this study.

1. A direct material balance control system utilizing distillate flow to control overhead composition gives better response when the distillation column is subjected to energy changes rather than flow changes. This should be true for all disturbances when both terminal compositions are under control with the reboiler duty used as one of the manipulated variables for composition control.
2. For changes in feed flow, an energy balance control scheme with the composition controlled by the reflux flow, gives superior control performance to the direct material balance control scheme.
3. The response of the distillation column was predicted using a material balance model in the linear state space form. Although the actual system is highly nonlinear, for control purposes the linearized model provides an adequate prediction even though the agreement with experimental open loop responses was not very good.
4. Feedforward and decoupling control matrices as proposed in this study have the advantage of allowing for feedback control tuning. The control scheme based on

a theoretical model can be implemented with on-line tuning to compensate for shortcomings in the model.

5. By eliminating algebraic loops the control scheme can be implemented with a digital computer and gives sufficient decoupling for regulatory action.
6. The use of a reduced model to find the control scheme gave a reasonable control performance. Restriction of the minimum dimension of the reduced model was dictated by the measurement noise rather than system dynamics. In this study the high noise level in the bottoms measurement made it necessary to retain two other states in addition to the terminal compositions.
7. Since the control matrices contain only proportional elements the continuous version of the control law should be possible to implement with conventional pneumatic or electronic instrumentation.

The following are some recommendations for future work.

1. Time delay and noise associated with the gas chromatograph used to measure the bottom composition are a major source of disturbance in the system. Improvement in the equipment is an area that requires some work. Possibilities include enlarging sample

size and speeding vaporization of the sample by increasing temperature or gas velocity in the sample area.

2. In addition to improving the measurement equipment a comparison of the use of a Kalman filter to condition the measurement value to exponential filtering would be relevant.
3. This system is also suitable for testing the application of a Smith Predictor to lessen the effect of the time delay associated with sampling.
4. A more general method of improving control would be the addition af a state estimation technique such as the steady state type proposed by Weber and Brosilow (50). This could be compared to the dynamic observer of Luenberger (23). Both methods would enable use of a tenth order concentration model based control law with temperature measurements used to estimate the states.
5. Model reference adaptive control because of its self tuning feature is suitable for use with a simple model. Also the continuous tuning feature would provide compensation for the nonlinearity of distillation columns. The pilot scale distillation column is suited to this type of test.

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NOMENCLATURE

A - model matrix

ADC - analog to digital converter for digital computer input

ARC - analyzer recording controller generally capacitance dynalog

B - bottoms flow (lb/min)

$\underline{\underline{B}}$ - control matrix

$\underline{\underline{C}}$ - disturbance matrix

$\underline{d}$ - disturbance vector $[F\ x_F\ h_F]^T$

$\underline{D}$ - Laplace transform of $\underline{d}$

D - distillate flow (lb/min)

DAC - digital to analog converter for digital computer output

DDC - Direct Digital Control - IBM control program

F - feed flow (lb/min)

$\underline{\underline{G}}$ - $(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}$

h - liquid enthalpy (BTU/lb)

$\underline{\underline{H}}$ - output matrix

H - vapor enthalpy (BTU/lb)

$\underline{\underline{I}}$ - identity matrix

I_j - interactive index

IAE - integral of the absolute error

IE - integral of the error

$\underline{\underline{K}}$ - control matrices

$\underline{\underline{L}}$ - $\underline{\underline{A}}_1 + \underline{\underline{A}}_3 \underline{\underline{G}} \underline{\underline{A}}_7$

L_i - liquid flow from stage i (lb/min)

$\underline{\underline{M}}$ - $\underline{\underline{A}}_2 + \underline{\underline{A}}_3 \underline{\underline{G}} \underline{\underline{A}}_8$

$\underline{\underline{P}}$ - $\underline{\underline{B}}_1 + \underline{\underline{A}}_3 \underline{\underline{G}} \underline{\underline{B}}_3$

$\underline{\underline{Q}}$ - $\underline{\underline{C}}_1 + \underline{\underline{A}}_3 \underline{\underline{G}} \underline{\underline{C}}_3$

q_i - heat loss from stage i (BTU/min)

Q_i - total heat transferred to stage i (BTU/min)

Q_R - reboiler load (BTU/min)

R - reflux flow (lb/min)

s - Laplace variable

S - steam flow to reboiler (lb/min)

S_{Li} - liquid draw from stage i (lb/min)

S_{Vi} - vapor draw from stage i (lb/min)

T_i - temperature of stage i

$\underline{u}$ - control vector $[Q_R \; D]^T$

$\underline{U}$ - Laplace transform of $\underline{u}$

$\underline{V}$ - feedback control vector

$\underline{\underline{W}}$ - discrete feedback matrix

W_{Li} - liquid holdup on stage i (lb)

W_{Vi} - vapor holdup on stage i (lb)

x_i - composition on stage i (wt. pct. MEOH)

Y_i - vapor composition on stage i (wt. pct. MEOH)

$\underline{z}$ - vector of states $[x_1,\ldots,x_{10}]^T$

$\underline{Z}$ - Laplace transform of $\underline{z}$

Greek Letters

$\underline{\underline{\phi}}$ - fundamental matrix

$\underline{\underline{\beta}}$ - discrete control matrix

$\underline{\underline{\Delta}}$ - discrete disturbance matrix

λ_{ij} - relative gain between variable i and variable j

Superscripts

$'$ - deviation variable

$-$ - steady state value

$\sim$ - approximate case

$\hat{}$ - discrete case

$\cdot$ - time derivative

Subscripts

F - feed

FF - feedforward

FB - feedback

P - matrix with main diagonal set to zero

d - diagonal matrix

DC - decoupling (for feedback)

UC - decoupling (for feedforward)

APPENDIX A

PHYSICAL SYSTEM DATA

A-1 Schematic of Distillation Column

Figure A-1 is a detailed schematic of the pilot scale distillation column. In addition to showing process streams the diagram shows the DDC loops and multiplexer points associated with variables along with the analog control elements on the column.

A-2 Typical Steady State Conditions

Table A-1 gives a listing of the variables as recorded during a steady state run.

In Figure A-2 some of the major variables are plotted.

A-3 Temperature and Enthalpy as a Function of Concentration

At constant pressure liquid enthalpy is a function of concentration. At one atmosphere the relation between enthalpy and concentration of liquid methanol-water is as shown in Figure A-3. The relation used to relate temperature and concentration is given in Figure A-4.

A-4 Methanol-Water Equilibrium Curve

The equilibrium curve for the methanol-water system is shown in Figure A-5. Also shown is the pseudo-equilibrium curve taken from experimental data obtained on the pilot scale distillation column.

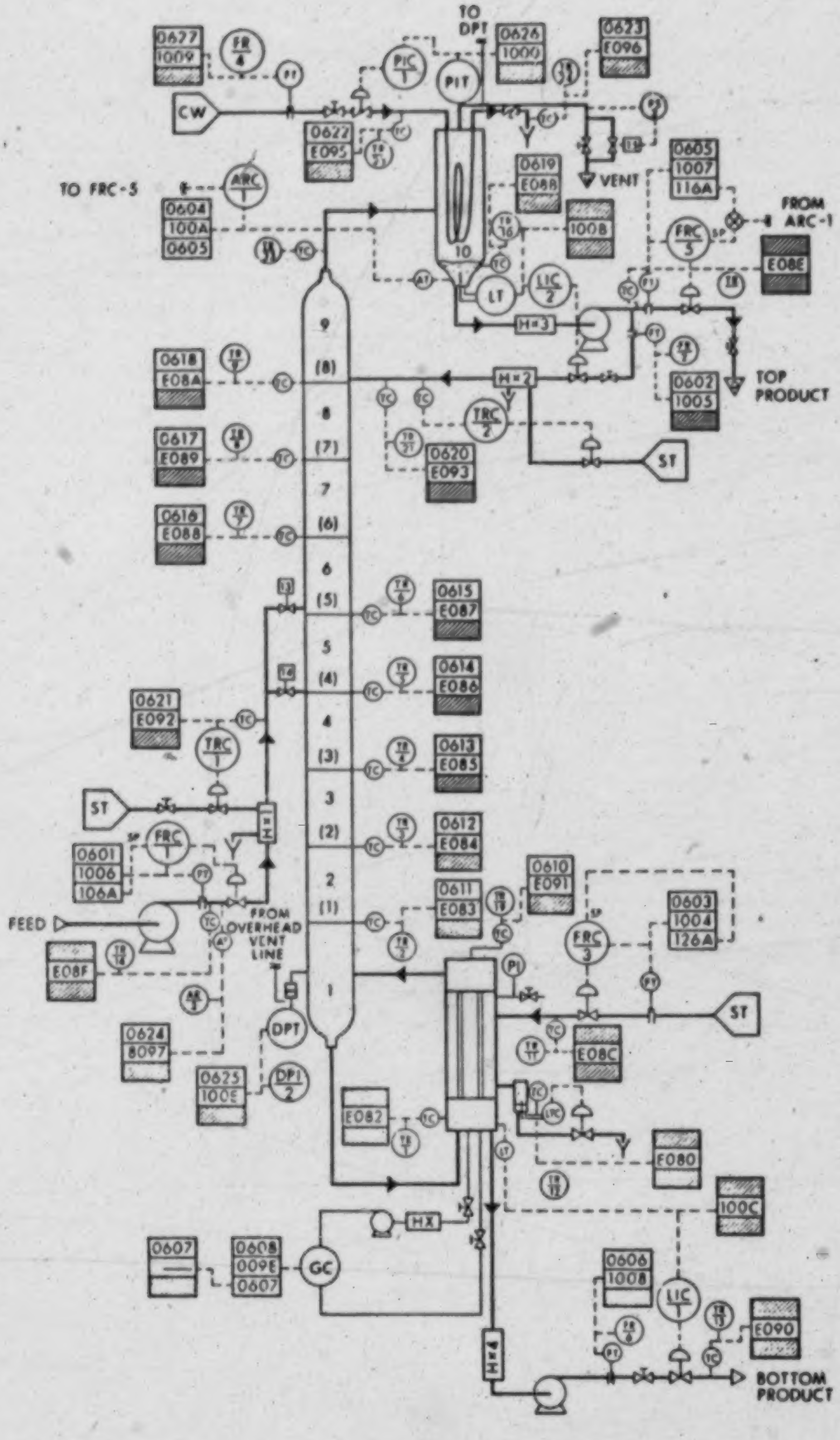

FIGURE A-1: A Detailed Schematic of the Pilot Scale Distillation Column.

LEGEND

LPID / ADC / DAC - indicates measurements available to IBM 1800
LPID - DDC loop identification code in hexidecimal
- crosshatching indicates no loop
ADC - multiplexer input address (in hexidecimal)
DAC - multiplexer output address (in hexidecimal)
- crosshatching indicated no output
loop is for data acquisition only

TYPE / n - indicates local analog equipment
TYPE - gives function
TYPE CODES
T - transmitter
R - recorder
C - controller
I - indicator
F - flow
A - concentration (analyzer)
T - temperature
P - pressure
DP - differential pressure
n - identification number

HXn - heater exchanger
n - identification number

GC - gas chromatograph

TC - thermocouple

UT - indicates utilities
UT - ST - 60 psig steam
- CW - cooling water

- solenoid valve
n - identification number

– – – - control lines

——— - process lines

TABLE A-1

STEADY STATE DATA
RUN NO SS 3 19/
10/73

FEED FLOW	2.400	LB/MIN	BOTTOM PROD	1.188	LB/MIN
REFLUX FLOW	1.936	LB/MIN	TOP PROD	1.197	LB/MIN
STEAM FLOW	1.851	LB/MIN	COOL WATER	37.700	LB/MIN
FEED PLATE	4		FEED COMP	50.20	WT P C
TOP PROD	97.36	WT P C	BOTTOMS COMP	3.00	WT P C
FEED INLET	161.2	DEG F	REFLUX INLET	140.4	DEG F
STEAM TEMP	229.7	DEG F	PRESSURE	1.4	IN H2O

MATERIAL BALANCE

	FLOW (LB/MIN)	COMP (WT PCT)	METHANOL (LB/MIN)	WATER (LB/MIN)
FEED	2.400	50.200	1.204	1.195
BOTTOM PRODUCT	1.188	3.000	0.035	1.152
TOP PRODUCT	1.197	97.369	1.165	0.031
CLOSURE ERROR-PC	-0.6		-0.3	-0.9

ENERGY BALANCE

	ENTHALPY IN (BTU/MIN)	ENTHALPY OUT (BTU/MIN)
COOLING WATER	2298.0	3834.7
REFLUX	193.2	198.5
TOP PRODUCT		122.7
FEED	336.4	
STEAM	2213.4	424.9
BOTTOM PRODUCT		248.4
TOTAL	5041.1	4829.3
HEAT LOSS		211.7

TABLE A-1 (Continued)

```
STEADY STATE CONDITIONS BASED ON  10 POINTS
         RUN NO SS 3 19/ 10/73

FEED FLOW    =  2.400  LB/MIN     DEV= 0.0061
REFLUX FLOW =  1.936  LB/MIN     DEV= 0.0101
STEAM FLOW  =  1.851  LB/MIN     DEV= 0.0100
BOTTOM PROD =  1.188  LB/MIN     DEV= 0.0118
TOP PROD    =  1.197  LB/MIN     DEV= 0.0094
COOL WATER  = 37.684  LB/MIN     DEV= 0.8833
TOP PROD    = 97.369  WT P C     DEV= 0.0721
BOTTOMS COMP=  3.000  WT P C     DEV= 0.0000
FEED COMP   = 50.200  WT P C     DEV= 0.0000
PRESSURE    =  1.458  IN H2O     DEV= 0.2332
COND LEVEL  =  5.105  PSIG       DEV= 0.0245
REB'R LEVEL =  9.953  PSIG       DEV= 0.0670
DIFF PRESS  = 10.887  PSIG       DEV= 0.2190
REBOILER TEM=  208.6  DEG F      DEV= 0.3651
PLATE 1 TEMP=  197.3  DEG F      DEV= 0.6236
PLATE 2 TEMP=  183.8  DEG F      DEV= 0.4533
PLATE 3 TEMP=  174.8  DEG F      DEV= 0.5041
PLATE 4 TEMP=  165.9  DEG F      DEV= 0.3855
PLATE 5 TEMP=  158.2  DEG F      DEV= 0.2252
PLATE 6 TEMP=  152.6  DEG F      DEV= 0.3405
PLATE 7 TEMP=  149.6  DEG F      DEV= 0.2700
PLATE 8 TEMP=  146.7  DEG F      DEV= 0.2850
COND TEMP   =  143.3  DEG F      DEV= 0.3259
STEAM TEMP  =  229.7  DEG F      DEV= 0.2839
COND'T TEMP =  227.2  DEG F      DEV= 0.3249
REFLUX FLOW =  124.1  DEG F      DEV= 0.3466
FEED FLOW   =   92.0  DEG F      DEV= 0.2312
BOTTOM FLOW =  103.4  DEG F      DEV= 0.2770
REB O'HEAD  =  206.6  DEG F      DEV= 0.3313
FEED INLET  =  161.2  DEG F      DEV= 0.2386
REFLUX INLET=  140.4  DEG F      DEV= 0.3302
COL O'HEAD  =  147.4  DEG F      DEV= 0.1900
WATER INLET =   61.0  DEG F      DEV= 0.2124
WATER OUTLET=  101.8  DEG F      DEV= 0.5435
```

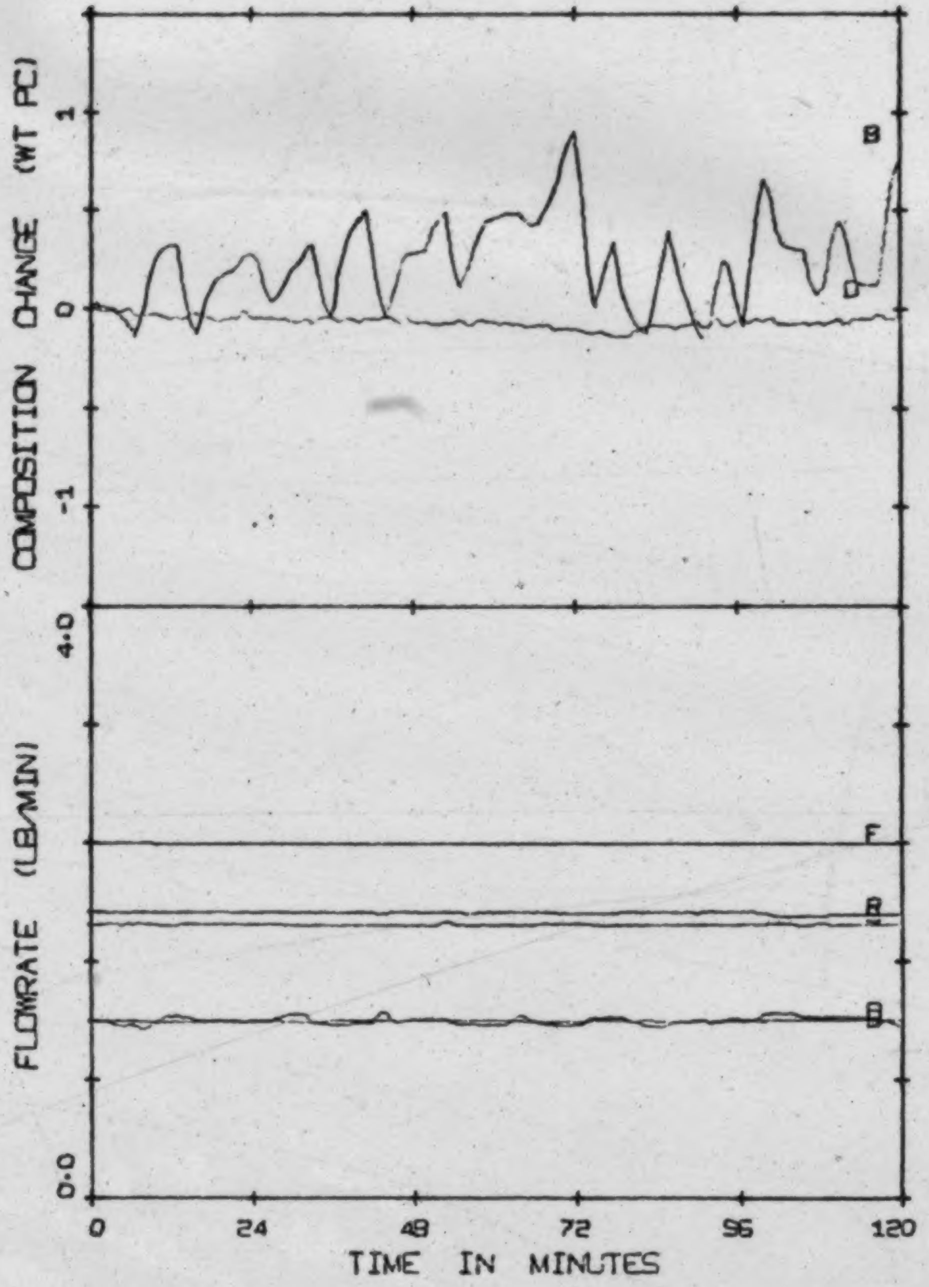

FIGURE A-2: Experimental Steady State Data.

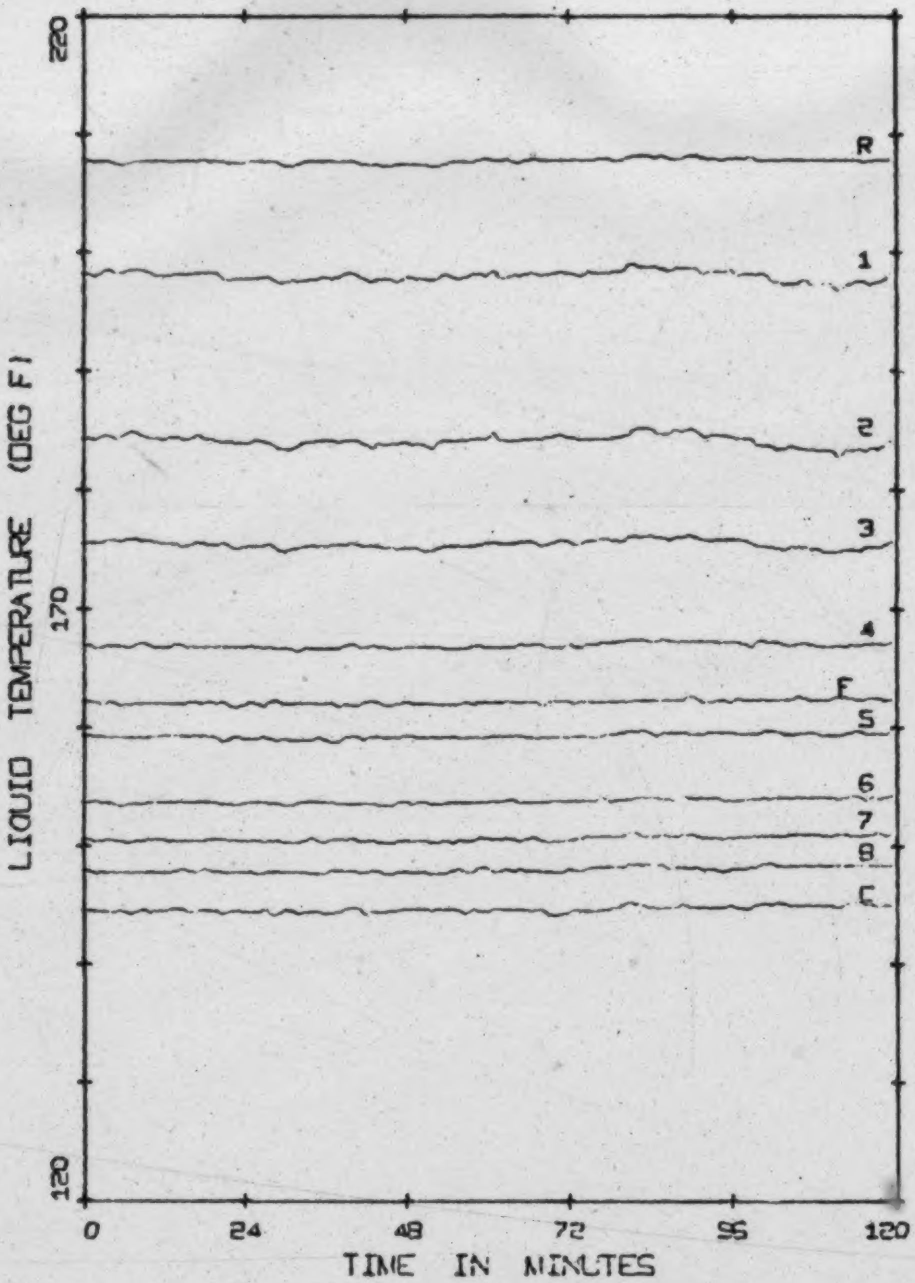

FIGURE A-2 Continued Experimental Steady State Data.

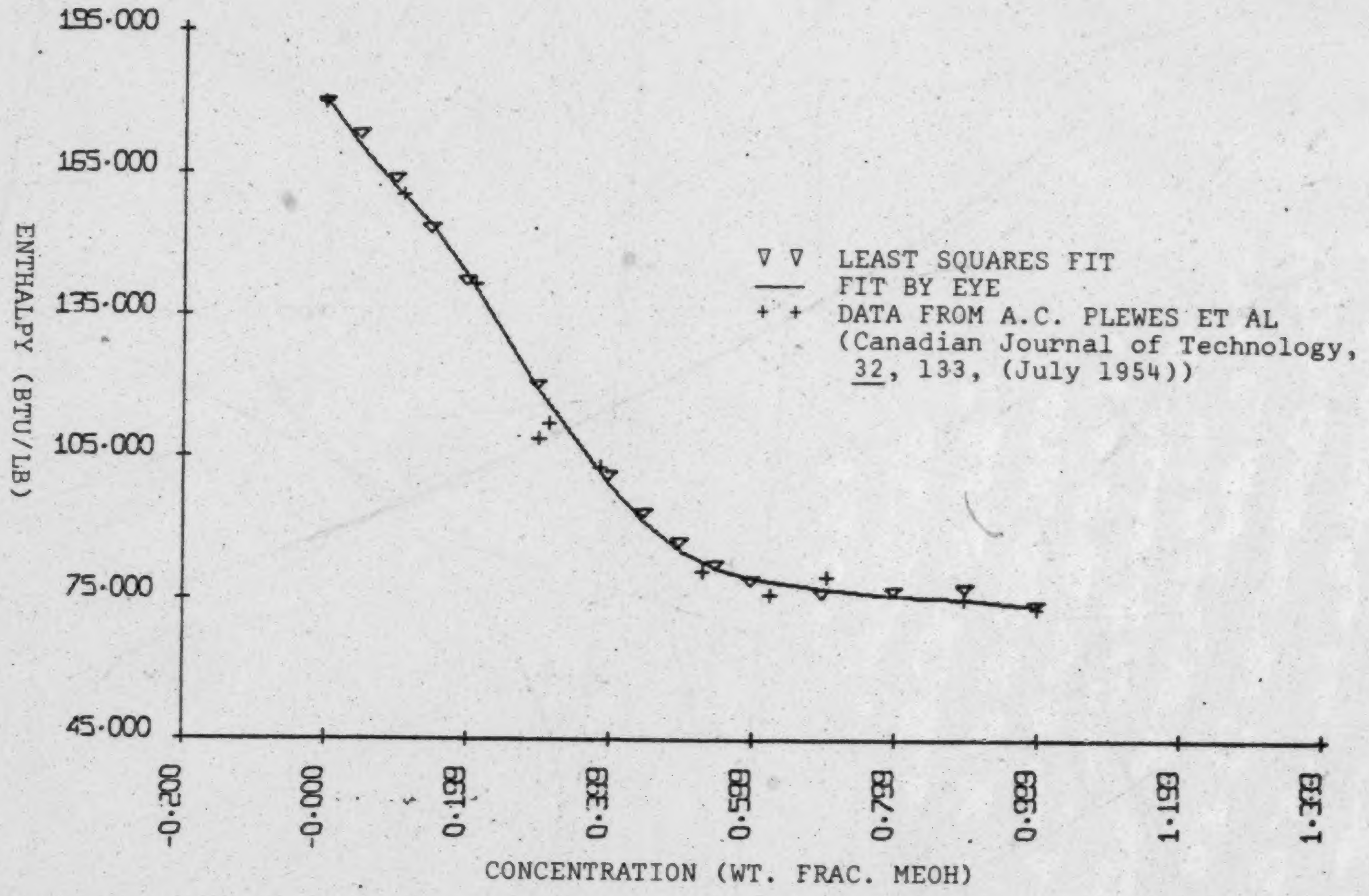

FIGURE A-3: Enthalpy Versus Concentration for Saturated Methanol-Water Solutions at One Atmosphere.

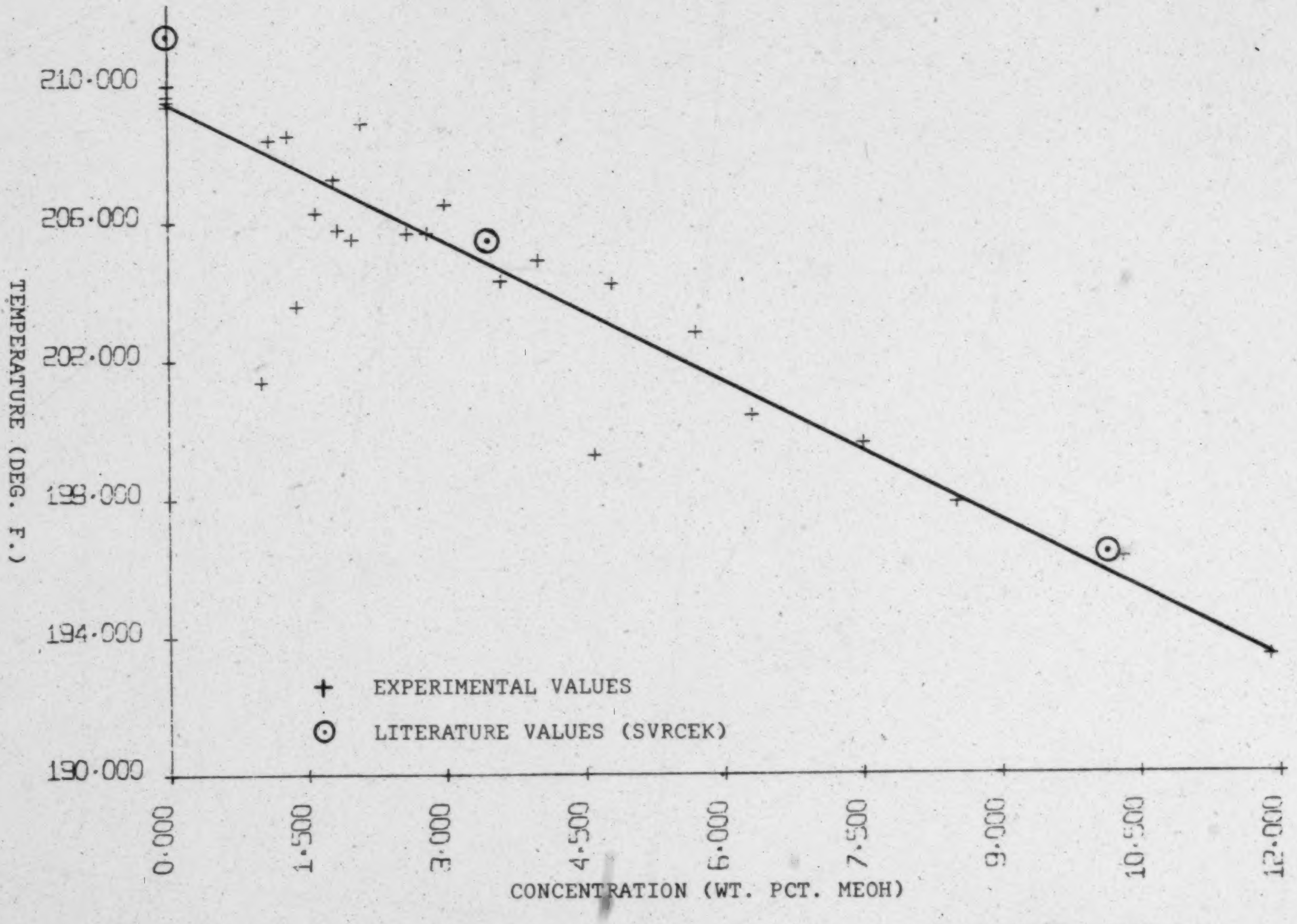

FIGURE A-4: Temperature versus Concentration For Saturated Methanol-Water at One Atmosphere.

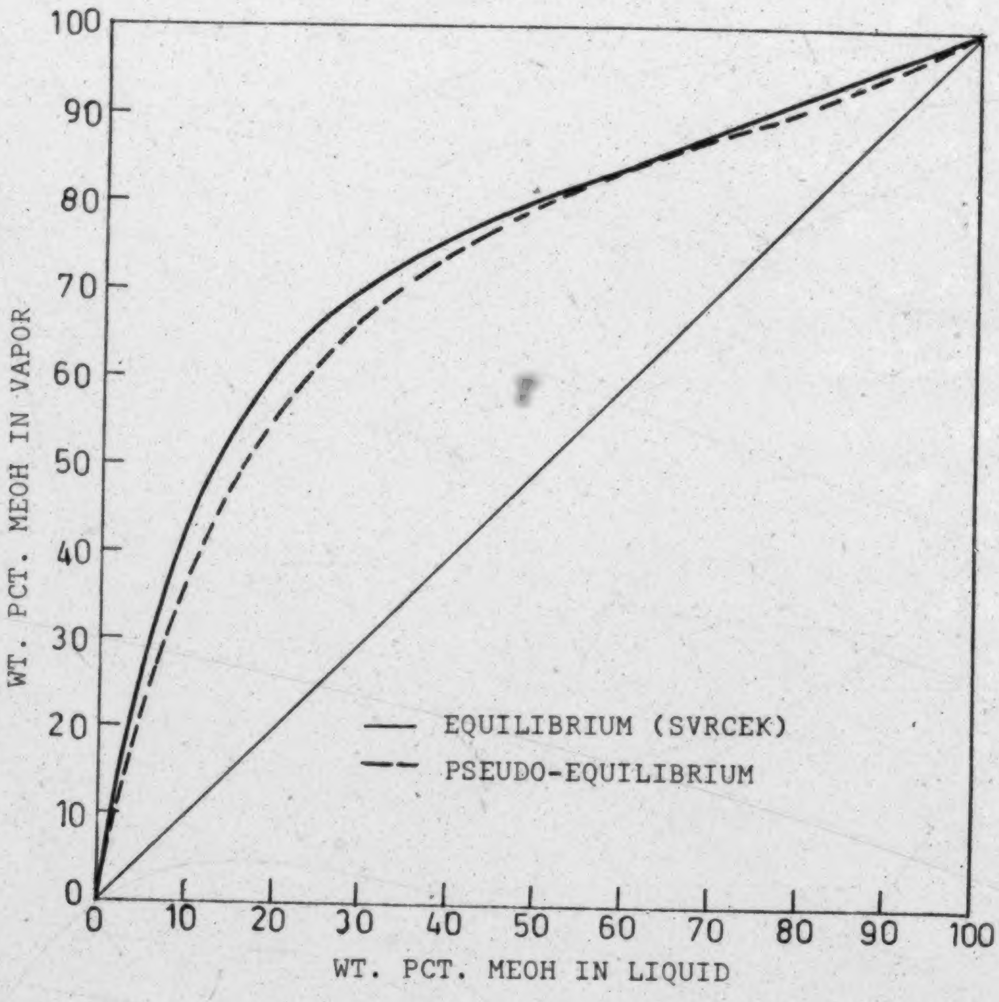

FIGURE A-5: Equilibrium For Methanol Water System At One Atmosphere.

APPENDIX B

MODEL EQUATIONS

B-1 Enthalpy Model

Equations for non-steady state enthalpy balances for the individual stages of the actual column are given as:

Reboiler;

$$V_{i-1} = 0$$

$$Q_i = Q_R$$

$$\dot{h}_1 = (h_2 L_2 - h_1 L_1 - H_1 V_1 + Q_R + q_1) \big/ (W_{L1} + B_{11}\, W_{V1}) \qquad \text{B-1}$$

where

$$V_1 = (Q_R + q_1) \big/ (H_1 - h_1) \qquad \text{B-2}$$

$$L_1 = F - D \qquad \text{B-3}$$

Trays 1 to 3; $i = 2\text{-}4$

$$\dot{h}_i = (h_{i+1}\, L_{i+1} + H_{i-1}\, V_{i-1} - h_i L_i - H_i V_i + Q_i) \big/ (W_{Li} + B_{1i}\, W_{Vi}) \qquad \text{B-4}$$

where

$$V_i = (Q_R + \sum_{j=1}^{i} q_j) \big/ (H_i - h_i) \qquad \text{B-5}$$

$$L_i = F - D + V_{i-1} \qquad \text{B-6}$$

Tray 4 (feed tray); $i = 5$

$$\dot{h}_5 = (h_6 L_6 + H_4 V_4 + h_F F - h_5 L_5 - H_5 V_5 + Q_5) \big/ (W_{L5} + B_{15}\, W_{V5}) \qquad \text{B-7}$$

(note that there is only one feed so the subscripts were dropped for quantities associated with it.)

$$V_5 = Q_R + \sum_{j=1}^{5} q_j + \Delta h_F^* F \qquad \text{B-8}$$

where $\Delta h_F^* = h_F^* - h_F$

and h_F^* - saturated feed enthalpy

$$L_5 = F - D + V_4 \qquad \text{B-9}$$

Trays 5 to 8; $i = 6$-9

$$\dot{h}_i = (h_{i+1} L_{i+1} + H_{i-1} V_{i-1} - h_i L_i - H_i V_i + q_i) \Big/ (W_{Li} + B_{1i} W_{Vi}) \qquad \text{B-10}$$

where $$V_i = \left(Q_R + \sum_{j=1}^{i} q_j + \Delta h_F^* F\right) \Big/ (H_i - h_i) \qquad \text{B-11}$$

$$L_i = V_{i-1} - D$$

Condenser; $i = 10$

$$\dot{h}_{10} = (H_9 V_9 - h_{10} (L_{10} + D)) \Big/ W_{L10} \qquad \text{B-13}$$

$$L_{10} = R = V_9 - D \qquad \text{B-14}$$

B-2 Concentration Model

The material balance equations can be written for the actual stages of the physical equipment. The expressions for liquid and vapor flow are the same as used in Appendix B-1. The resulting equations are:

Reboiler; $i = 1$

$$\dot{x}_i = (x_2 L_2 - x_1 L_1 - Y_1 V_1) \Big/ W_{L1} \qquad \text{B-15}$$

Trays 1 to 3; i = 2-4

$$\dot{x}_i = (x_{i+1} L_{i+1} + Y_{i-1} V_{i-1} - x_i L_i - Y_i V_i) / W_{Li} \qquad \text{B-16}$$

Tray 4; i = 5 (feed tray)

$$\dot{x}_5 = (x_6 L_6 + Y_4 V_4 + x_F F - x_5 L_5 - Y_5 V_5) / W_{L5} \qquad \text{B-17}$$

Trays 5 to 8; i = 6-9

$$\dot{x}_i = (x_{i+1} L_{i+1} + Y_{i-1} V_{i-1} - x_i L_i - Y_i V_i) / W_{Li} \qquad \text{B-18}$$

Condenser; i=10

$$\dot{x}_{10} = (Y_9 V_9 - x_{10} (L_{10} - D)) / W_{L10} \qquad \text{B-19}$$

B-3 Linearization of the Concentration Model

In order to obtain a linear model the linear terms of a Taylor series expansion of the concentration model were taken. The general equation is

$$\dot{x}_i = (x_{i+1} L_{i+1} + Y_{i-1} V_{i-1} + x_{Fi} F_i - x_i L_i - Y_i V_i) / W_{Li} \qquad \text{B-20}$$

The linear version is given by

$$\dot{x}_i = (\bar{x}_{i+1} L'_{i+1} + x'_{i+1} \bar{L}_{i+1} + \bar{Y}_{i-1} V'_{i-1} + Y'_{i-1} \bar{V}_{i-1} + \bar{x}_{Fi} F'_i + x'_{Fi} \bar{F}_i - \bar{x}_i L'_i - x'_i \bar{L}_i - \bar{Y}_i V'_i - Y'_i \bar{V}_i) / \bar{W}_{Li} \qquad \text{B-21}$$

The linearized vapor flows are given by

$$V_i' = Q_R \Big/ (\bar{H}_i - \bar{h}_i) \quad \text{for } i = 1\text{-}4$$

$$V_i' = (Q_R' + F_i' \bar{h}_{Fi} + \bar{F}_i h_{Fi}') \Big/ (\bar{H}_i - \bar{h}_i) \quad \text{for } i = 5\text{-}10$$

In order to obtain a linear relationship between x and Y use

$$Y_i' = C_{1i} x_i' \qquad \text{B-22}$$

where

$$C_{1i} = \left.\frac{\delta Y_i}{\delta x_i}\right|_{\bar{x}_i}$$

The liquid flow can be obtained from mass balances.

$$L_i = B + V_{i-1} \quad \text{for } i = 1\text{-}4 \qquad \text{B-23}$$

$$L_i = B + V_{i-1} - F \quad \text{for } i = 5\text{-}9 \qquad \text{B-24}$$

$$L_{10} = V_9 - D \qquad \text{B-25}$$

If this model is put in the state space form given by

$$\dot{\underline{z}} = \underline{\underline{A}}\, \underline{z} + \underline{\underline{B}}\, \underline{u} + \underline{\underline{C}}\, \underline{d} \qquad \text{B-26}$$

then the elements of $\underline{\underline{A}}$ are given by:

Main diagonal

$$a_{i,i} = (-\bar{L}_i - \bar{V}_i C_{1i}) \Big/ \bar{W}_{Li} \qquad \text{B-27}$$

Super diagonal

$$a_{i,i+1} = \bar{L}_{i+1} \Big/ \bar{W}_{Li} \qquad \text{B-28}$$

Sub-diagonal

$$a_{i,i-1} = (\bar{V}_{i-1} C_{1i-1}) \Big/ \bar{W}_{Li} \qquad \text{B-29}$$

The remaining elements are zero.

Letting

$$r_i = (\bar{Y}_{i-1} - \bar{x}_i) \ (\bar{H}_i - \bar{h}_i) + (\bar{x}_{i+1} - \bar{Y}_i) \ (\bar{H}_i - \bar{h}_i) \qquad \text{B-30}$$

the elements of $\underline{\underline{B}}$ are given by:

$$b_{i,1} = r_i \Big/ \bar{W}_{Li} \qquad \text{for } i = 1\text{-}10 \qquad \text{B-31}$$

$$b_{i,2} = (\bar{x}_i - \bar{x}_{i+1}) \Big/ \bar{W}_{Li} \qquad \text{for } i = 1\text{-}10 \qquad \text{B-32}$$

and the elements of $\underline{\underline{C}}$ are given by:

$$c_{i,1} = (\bar{x}_{i+1} - \bar{x}_i) \Big/ \bar{W}_{Li} \qquad i = 1\text{-}4 \qquad \text{B-33}$$

$$c_{i,2} = c_{i,3} = 0 \qquad i = 1\text{-}4 \qquad \text{B-34}$$

$$c_{5,1} = (\bar{x}_F - \bar{x}_5 + \bar{h}_F \, r_5) \Big/ \bar{W}_{L5} \qquad \text{B-35}$$

$$c_{5,2} = \bar{F} \Big/ \bar{W}_{L5} \qquad \text{B-36}$$

$$c_{5,3} = r_5 \, \bar{F} \Big/ \bar{W}_{L5} \qquad \text{B-37}$$

$$c_{i,1} = r_i \, \bar{h}_F \Big/ \bar{W}_{Li} \qquad i = 6\text{-}9 \qquad \text{B-38}$$

$$c_{i,2} = 0 \qquad i = 6\text{-}9 \qquad \text{B-39}$$

$$c_{i,3} = r_i \, F \Big/ W_{Li} \qquad i = 6\text{-}9 \qquad \text{B-40}$$

$$c_{10,1} = c_{10,2} = c_{10,3} = 0 \qquad \text{B-41}$$

B-4 Values for Concentration Model

Table B-1 contains numerical values for the linear state space form of the concentration model using the steady state values as given in Table B-2. The form of the model is as shown in equations 3.10 and 3.11.

Table B-3 gives the eigenvalues of the model matrix $\underline{A}$.

TABLE B-1

LINEAR CONCENTRATION MODEL MATRICES

MATRIX $\underline{\mathbf{A}}$

$$\begin{bmatrix}
-0.6424 & 0.2338 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5.286 & -4.280 & 2.462 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.964 & -3.293 & 2.494 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.8976 & -3.149 & 2.472 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.6962 & -2.967 & 0.9623 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5795 & -1.586 & 1.025 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.6455 & -1.778 & 1.119 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.7533 & -1.982 & 1.242 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8384 & -2.177 & 1.312 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4357 & -1.084
\end{bmatrix}$$

MATRIX $\underline{\mathbf{B}}$

$$\begin{bmatrix}
-6.114\text{E-}6 & -8.054\text{E-}3 \\
-9.453\text{E-}5 & -8.1222 \\
-6.485\text{E-}5 & -0.08168 \\
-2.913\text{E-}5 & -0.03468 \\
1.421\text{E-}4 & -0.1050 \\
3.997\text{E-}5 & -0.06385 \\
2.340\text{E-}5 & -0.03704 \\
2.029\text{E-}5 & -0.03168 \\
-8.710\text{E-}6 & -0.01345 \\
0 & 0
\end{bmatrix}$$

MATRIX $\underline{\mathbf{C}}$

$$\begin{bmatrix}
0.0081 & 0 & 0 \\
0.1222 & 0 & 0 \\
0.0817 & 0 & 0 \\
0.3468 & 0 & 0 \\
-0.0576 & 1.415 & 3.41\text{E-}4 \\
-4.00\text{E-}4 & 0 & 9.59\text{E-}5 \\
-2.34\text{E-}4 & 0 & 5.62\text{E-}5 \\
-2.03\text{E-}5 & 0 & 4.87\text{E-}5 \\
-8.71\text{E-}5 & 0 & 2.09\text{E-}5 \\
0 & 0 & 0
\end{bmatrix}$$

TABLE B-2

STEADY STATE DATA
RUN NO .05D OL
8/7/73 0

FEED FLOW	2.402 LB/MIN	BOTTOM PROD	1.330 LB/MIN
REFLUX FLOW	2.215 LB/MIN	TOP PROD	1.058 LB/MIN
STEAM FLOW	1.843 LB/MIN	COOL WATER	41.897 LB/MIN
FEED PLATE	4	FEED COMP	45.60 WT P C
TOP PROD	97.22 WT P C	BOTTOMS COMP	6.18 WT P C
FEED INLET	162.4 DEG F	REFLUX INLET	138.8 DEG F
STEAM TEMP	229.3 DEG F	PRESSURE	1.3 IN H2O

MATERIAL BALANCE

	FLOW (LB/MIN)	COMP (WT PCT)	METHANOL (LB/MIN)	WATER (LB/MIN)
FEED	2.402	45.600	1.095	1.307
BOTTOM PRODUCT	1.330	6.180	0.082	1.248
TOP PRODUCT	1.058	97.228	1.029	0.029
CLOSURE ERROR-PC	-0.5		1.4	-2.2

ENERGY BALANCE

	ENTHALPY IN (BTU/MIN)	ENTHALPY OUT (BTU/MIN)
COOLING WATER	2780.4	4300.0
REFLUX	217.8	229.9
TOP PRODUCT		109.9
FEED	344.4	
STEAM	2204.4	422.3
BOTTOM PRODUCT		273.3
TOTAL	5547.1	5335.6
HEAT LOSS		211.4

TABLE B-2 (Continued)

```
STEADY STATE CONDITIONS BASED ON  10 POINTS
         RUN NO .05D OL  8/7/73 0

FEED FLOW    =  2.402  LB/MIN    DEV= 0.0041
REFLUX FLOW =   2.215  LB/MIN    DEV= 0.0086
STEAM FLOW   =  1.843  LB/MIN    DEV= 0.0146
BOTTOM PROD =   1.330  LB/MIN    DEV= 0.0083
TOP PROD     =  1.058  LB/MIN    DEV= 0.0096
COOL WATER   = 41.897  LB/MIN    DEV= 0.5104
TOP PROD     = 97.228  WT P C    DEV= 0.0893
BOTTOMS COMP=   6.180  WT P C    DEV= 0.0000
FEED COMP    = 45.600  WT P C    DEV= 0.0000
PRESSURE     =  1.344  IN H2O    DEV= 0.2553
COND LEVEL   =  5.178  PSIG      DEV= 0.0263
REB'R LEVEL =   9.812  PSIG      DEV= 0.0622
DIFF PRESS   = 11.734  PSIG      DEV= 0.4645
REBOILER TEM=   206.3  DEG F     DEV= 0.3709
PLATE 1 TEMP=   188.4  DEG F     DEV= 0.6800
PLATE 2 TEMP=   175.6  DEG F     DEV= 0.2838
PLATE 3 TEMP=   169.8  DEG F     DEV= 0.3689
PLATE 4 TEMP=   166.2  DEG F     DEV= 0.2500
PLATE 5 TEMP=   157.9  DEG F     DEV= 0.2444
PLATE 6 TEMP=   152.4  DEG F     DEV= 0.2863
PLATE 7 TEMP=   149.5  DEG F     DEV= 0.2267
PLATE 8 TEMP=   147.1  DEG F     DEV= 0.3343
COND TEMP    =  144.6  DEG F     DEV= 0.2541
STEAM TEMP   =  229.3  DEG F     DEV= 0.2473
COND'T TEMP =   226.8  DEG F     DEV= 0.2610
REFLUX FLOW =   126.1  DEG F     DEV= 0.2582
FEED FLOW    =   96.4  DEG F     DEV= 0.2514
BOTTOM FLOW =   110.0  DEG F     DEV= 0.2132
REB O'HEAD   =  204.3  DEG F     DEV= 0.1937
FEED INLET   =  162.4  DEG F     DEV= 0.3516
REFLUX INLET=   138.8  DEG F     DEV= 0.3837
COL O'HEAD   =  147.7  DEG F     DEV= 0.3395
WATER INLET =    66.4  DEG F     DEV= 0.2037
WATER OUTLET=   102.7  DEG F     DEV= 0.3456
```

TABLE B-3

EIGENVALUES OF CONCENTRATION MODEL MATRIX

	EIGENVALUE
1	-6.495
2	-4.500
3	-3.523
4	-2.769
5	-2.278
6	-1.150
7	-0.973
8	-0.652
9	-0.244
10	-0.055

APPENDIX C

SAMPLE DERIVATION OF CONTROL LAW FROM THIRD ORDER MODEL

In this section a control scheme is derived for a general third order linear state space model. Also a simplified control scheme is developed for comparison.

The continuous state space model can be represented by

$$\dot{\underline{z}} = \underline{\underline{A}}\,\underline{z} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{C}}\,\underline{d} \qquad \text{C-1}$$

This can be put in the linear pertubated form and the Laplace transform taken to give

$$s\,\underline{Z} = \underline{\underline{A}}\,\underline{Z} + \underline{\underline{B}}\,\underline{U} + \underline{\underline{C}}\,\underline{D} \qquad \text{C-2}$$

where

$\underline{Z}$ - $[z_1\ z_2\ z_3]^T$ - vector of the states

$\underline{U}$ - $[u_1\ u_2]^T$ - vector of the manipulated inputs

D - d_1 or d - disturbance input

If $a_{i,j}$ - element of $\underline{\underline{A}}$

$b_{i,j}$ - element of $\underline{\underline{B}}$

and $c_{i,j}$ - element of $\underline{\underline{C}}$

then the system represented by equation C-2 can be represented by the signal flow graph in Figure C-1.

If z_1 and z_2 are to be controlled using u_1 and u_2 respectively, feedforward and decoupling control action can be obtained by applying equation 4.29 to get

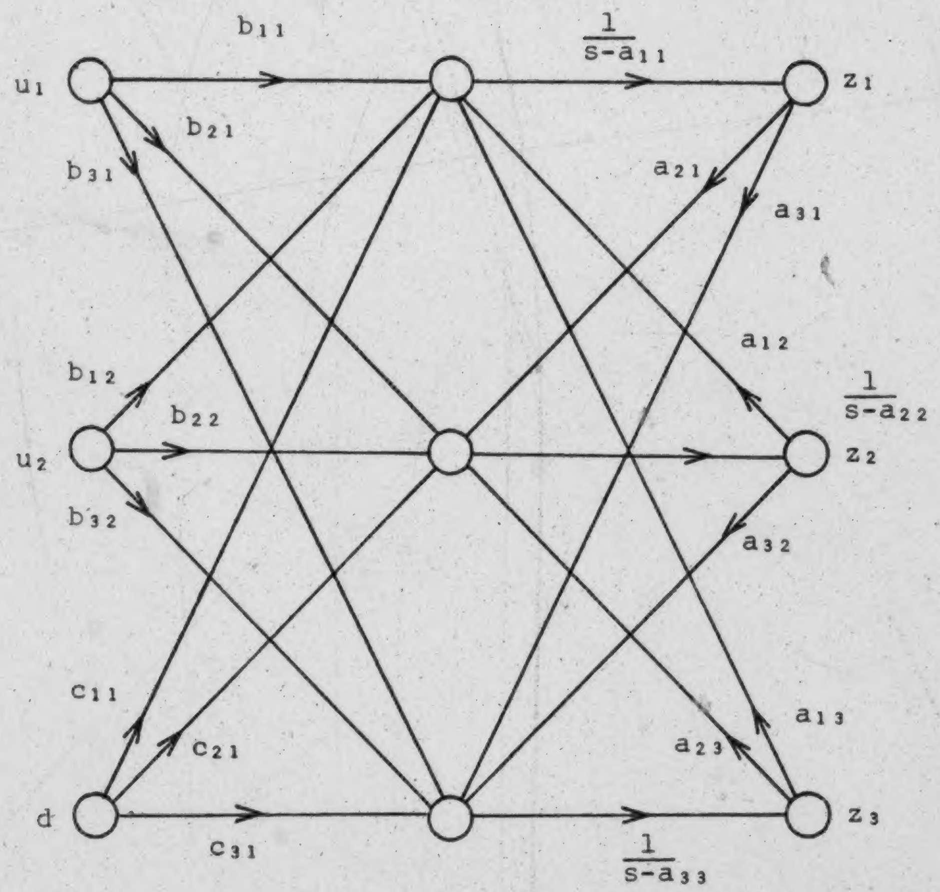

FIGURE C-1: Signal Flow Graph of System Equations.

$$u_1 = -(a_{1,2}z_2 + b_{1,2}u_2 + a_{1,3}z_3 + c_{1,1}d) / b_{1,1} \qquad \text{C-3}$$

$$u_2 = -(a_{2,1}z_1 + b_{2,1}u_1 + a_{2,3}z_3 + c_{2,1}d) / b_{2,2} \qquad \text{C-4}$$

This is shown by a signal flow graph in Figure C-2.

To obtain a solution these must be solved simultaneously for u_1 and u_2. This is easily done by analog elements but for implementation on a discrete system such as a digital computer an iterative numerical solution is necessary. Another approach is to use the analytic solution except that this does not allow direct decoupling of the manipulated variables as shown below. This becomes important when feedback is to be added to the control function.

To illustrate this let $\underline{V}(\underline{z})$ be the addition to $\underline{U}$ to provide feedback action. Equations C-3 and C-4 become

$$u_1 = -(a_{1,2}z_2 + b_{1,2}u_2 + a_{1,3}z_3 + c_{1,1}d) / b_{1,1} + v_1(z_1) \qquad \text{C-5}$$

$$u_2 = -(a_{2,1}z_1 + b_{2,1}u_1 + a_{2,3}z_3 + c_{2,1}d) / b_{2,2} + v_2(z_2) \qquad \text{C-6}$$

where v_1 and v_2 are constant at any time T. Solving equations C-5 and C-6 for u_1 and u_2 is possible analytically but is not practical because the resulting function for the manipulated inputs are functions of both feedback variables v_1 and v_2. If these are to be retained for tuning on-line they cannot be incorporated within the function.

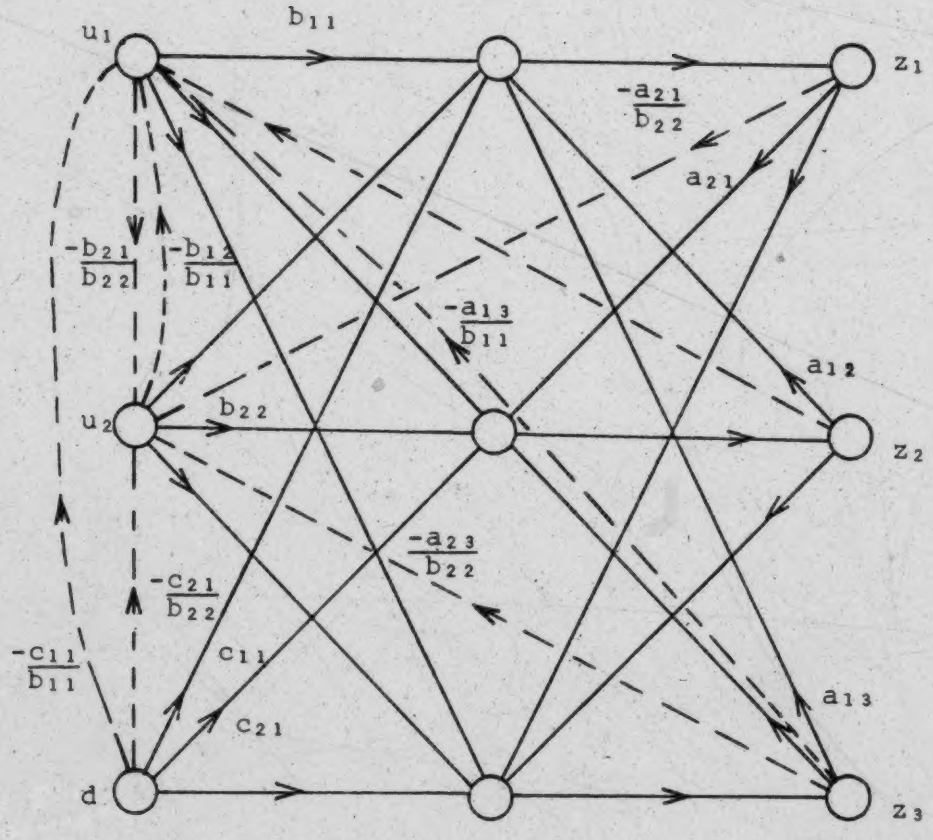

FIGURE C-2: Signal Flow Graph Showing Control Relationships

The approximate solution from equation 4.30 yeilds the simplified feedforward and decoupling control function given by

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} 0 & a_{1,2} & a_{1,3} \\ a_{2,1} & 0 & a_{2,3} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} c_{1,1} \\ c_{2,1} \end{bmatrix} d \qquad \text{C-7}$$

which has the form

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_1(z_1, z_2, z_3, d) \\ f_2(z_1, z_2, z_3, d) \end{bmatrix} \qquad \text{C-8}$$

The signal flow graph of this is shown in Figure C-3. Note that the algebraic loop between u_1 and u_2 has been eliminated. The feedback $\underline{V}$ is not decoupled at the inputs but must rely on decoupling initiated from the controlled states.

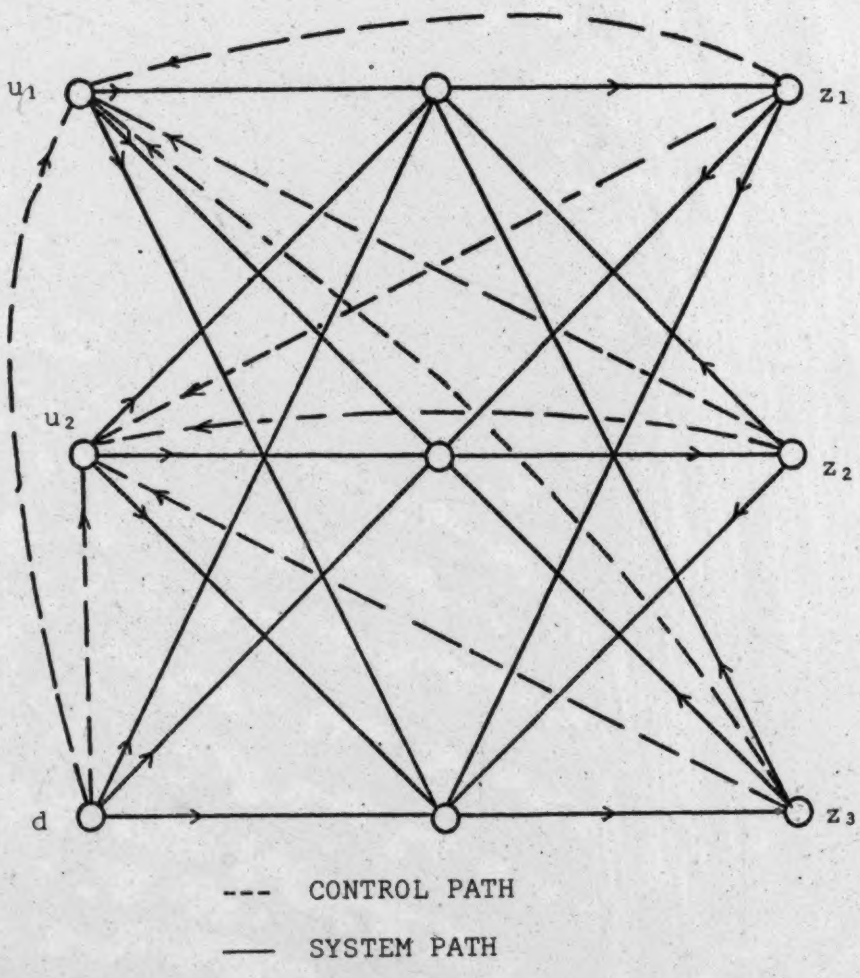

FIGURE C-3: Signal Flow Graph Representing Modified Control Equations With the System.

APPENDIX D

CONTROL MATRICES

The control matrices used in the simulations are given in the following figures along with the discrete model matrices.

First, the tenth order continuous control matrices are given in Figure D-1.

Figure D-2 shows the tenth order discrete model. It should be noted that the input matrix $\underline{\beta}$ shown here contains the coefficients of both control and disturbance inputs. The corresponding input vector is given by

$$\underline{u} = [F \; x_F \; h_F \; Q_R \; D] \tag{D-1}$$

Figure D-3 shows the tenth order control matrices.

Figures D-4 and D-5 contain the fourth order reduced model and control matrices.

Figures D-6 and D-7 give the second order reduced discrete model and control matrices.

Both the fourth and second order model input matrices contain coefficients for control and disturbance inputs and the corresponding input vector is given by equation D-1.

It should be noted that all the model and control matrices shown here are based on the tenth order concentration model presented in Appendix B.

MATRIX $\underline{\underline{K}}_{DC}$

$$\begin{bmatrix} 0 & 2.06E4 & 0 & 0 & 0 & 0 & 0 & -4.43E4 & 0 & -6.94E4 \\ 0 & 1.34E1 & 0 & 0 & 0 & 0 & 0 & 3.36E1 & 0 & 5.27E1 \end{bmatrix}$$

MATRIX $\underline{\underline{K}}_{FF}$

$$\begin{bmatrix} 7.16E1 & 0 & -1.11E0 \\ 4.57E-2 & 0 & 8.39E-4 \end{bmatrix}$$

FIGURE D-1: Continuous Linear Tenth Order Feedforward and Decoupling Matrices

MATRIX $\underline{\underline{\phi}}$

$$\begin{bmatrix}
6.93E-1 & 5.49E-2 & 4.05E-2 & 2.15E-2 & 1.30E-2 & 2.80E-3 & 5.09E-4 & 8.32E-5 & 1.27E-5 & 1.59E-6 \\
1.24E0 & 1.82E-1 & 2.16E-1 & 2.01E-1 & 1.44E-1 & 4.34E-2 & 1.01E-2 & 2.01E-3 & 3.62E-4 & 6.82E-5 \\
7.30E-1 & 1.72E-1 & 2.55E-1 & 3.00E-1 & 2.61E-1 & 1.00E-1 & 2.79E-2 & 6.40E-3 & 1.31E-3 & 2.78E-4 \\
1.63E-1 & 5.77E-2 & 1.07E-1 & 1.87E-1 & 2.39E-1 & 1.42E-1 & 5.21E-2 & 1.47E-2 & 3.54E-3 & 8.79E-4 \\
2.37E-2 & 1.16E-2 & 2.64E-2 & 6.73E-2 & 1.43E-1 & 1.60E-1 & 8.23E-2 & 2.94E-2 & 8.47E-3 & 2.51E-3 \\
3.09E-3 & 2.12E-3 & 6.12E-3 & 2.41E-2 & 9.65E-2 & 3.26E-1 & 2.58E-1 & 1.23E-1 & 4.35E-2 & 1.58E-2 \\
3.59E-4 & 3.09E-4 & 1.07E-3 & 5.57E-3 & 3.12E-2 & 1.62E-1 & 3.21E-1 & 2.53E-1 & 1.25E-1 & 6.21E-2 \\
3.88E-5 & 4.14E-5 & 1.66E-4 & 1.05E-3 & 7.51E-3 & 5.19E-2 & 1.70E-1 & 3.00E-1 & 2.35E-1 & 1.76E-1 \\
3.99E-6 & 5.04E-6 & 2.28E-5 & 1.72E-4 & 1.46E-3 & 1.25E-2 & 5.69E-2 & 1.59E-1 & 2.47E-1 & 3.38E-1 \\
1.66E-7 & 3.15E-7 & 1.61E-6 & 1.42E-5 & 1.44E-4 & 1.50E-3 & 9.36E-3 & 3.95E-2 & 1.12E-1 & 4.16E-1
\end{bmatrix}$$

MATRIX $\underline{\underline{\beta}}$

$$\begin{bmatrix}
1.35E-2 & 5.56E-3 & 1.41E-6 & -9.97E-6 & -1.42E-2 \\
6.99E-2 & 9.37E-2 & 2.41E-5 & -4.76E-5 & -8.17E-2 \\
6.27E-2 & 2.34E-1 & 6.07E-5 & -3.21E-5 & -9.26E-2 \\
1.22E-2 & 3.82E-1 & 1.01E-4 & 1.95E-5 & -6.26E-2 \\
-1.73E-2 & 5.39E-1 & 1.47E-4 & 5.72E-5 & -5.66E-2 \\
-4.64E-3 & 1.26E-1 & 9.92E-5 & 4.06E-5 & -5.59E-2 \\
-1.07E-3 & 2.34E-2 & 6.01E-5 & 2.49E-5 & -3.77E-2 \\
-3.10E-4 & 3.89E-3 & 4.09E-5 & 1.70E-5 & -2.64E-2 \\
-1.00E-4 & 5.76E-4 & 1.90E-5 & 7.93E-6 & -1.23E-2 \\
-1.43E-5 & 4.42E-5 & 3.04E-6 & 1.27E-6 & -1.97E-3
\end{bmatrix}$$

FIGURE D-2: Tenth Order Discrete Model Matrices (T = 1 Min.)

MATRIX $\hat{\underline{K}}_{DC}$

$$\begin{bmatrix} -6.26E-2 & 2.87E3 & 2.12E3 & 1.31E3 & 6.24E2 & -4.19E2 & -3.50E3 & -1.49E4 & -4.23E4 & 8.33E-2 \\ 4.41E-5 & 1.85E0 & 1.37E0 & 8.52E-1 & 4.76E-1 & 4.93E-1 & 2.50E0 & 1.05E1 & 2.98E1 & 5.37E-5 \end{bmatrix}$$

MATRIX $\hat{\underline{K}}_{FF}$

$$\begin{bmatrix} 7.10E2 & 2.74E2 & -1.07E0 \\ 4.51E-1 & 1.99E-1 & 8.56E-4 \end{bmatrix}$$

FIGURE D-3: Tenth Order Discrete Control Matrices (T = 1 Min.)

MATRIX $\underline{\phi}$

$$\begin{bmatrix} 5.61E-1 & 1.53E-1 & 1.07E-1 & -1.48E-1 \\ 3.10E-1 & 8.17E-1 & 1.01E0 & -1.37E0 \\ -4.28E-2 & 1.87E-2 & 9.84E-1 & -4.04E-1 \\ -7.53E-3 & 3.27E-3 & 2.67E-1 & 2.66E-1 \end{bmatrix}$$

MATRIX $\underline{B}$

$$\begin{bmatrix} 1.07E-2 & 2.80E-2 & 5.91E-6 & -6.69E-6 & -1.34E-2 \\ 4.85E-2 & 2.85E-1 & 6.31E-5 & -2.11E-5 & -7.80E-2 \\ -1.26E-3 & 5.02E-3 & 1.86E-5 & 8.54E-6 & -1.08E-2 \\ -2.13E-4 & 6.60E-4 & 2.90E-5 & 1.35E-6 & -1.67E-3 \end{bmatrix}$$

FIGURE D-4: Fourth Order Discrete Model Matrices
(Reduced from Tenth Order by Modal Reduction)

MATRIX $\hat{\underline{K}}_{DC}$

$$\begin{bmatrix} 3.08E3 & 7.49E3 & 6.19E3 & 2.05E4 \\ -1.54E0 & 7.67E0 & 4.90E0 & -21.3E1 \end{bmatrix}$$

MATRIX $\hat{\underline{K}}_{FF}$

$$\begin{bmatrix} 7.09E0 & 1.25E3 & -9.97E-1 \\ 4.44E-1 & 1.46E0 & 9.38E-4 \end{bmatrix}$$

FIGURE D-5: Fourth Order Discrete Control Matrices
(T = 1 Min.)

MATRIX $\underline{\Phi}$

$$\begin{bmatrix} 9.43E-1 & 2.45E-1 \\ 2.13E-3 & 7.87E-1 \end{bmatrix}$$

MATRIX $\underline{\underline{\beta}}$

$$\begin{bmatrix} 1.65E-2 & 8.91E-2 & 2.38E-5 & -5.87E-6 & -2.83E-2 \\ -3.24E-4 & 5.97E-3 & 1.03E-5 & 4.32E-6 & -6.19E-3 \end{bmatrix}$$

FIGURE D-6: Second Order Discrete Model Matrices
(Reduced from Tenth Order by Modal Reduction)
(T = 1 Min.)

MATRIX $\underline{\underline{\hat{K}}}_{DC}$

$$\begin{bmatrix} -3.81E2 & 9.57E3 \\ 7.90E-2 & 6.67E0 \end{bmatrix}$$

MATRIX $\underline{\underline{\hat{K}}}_{FF}$

$$\begin{bmatrix} 7.00E2 & 2.41E3 & -9.11E-1 \\ 4.36E-1 & 2.64E0 & 1.03E-3 \end{bmatrix}$$

FIGURE D-7: Second Order Discrete Control Matrices
(T = 1 Min.)

APPENDIX E

ON-LINE COMPUTER PROGRAMS

A series of programs run on the IBM 1800 computer enable control and data acquisition and storage for testing on the pilot scale distillation column. These programs were originated by various workers with some modifications. Only the present usage of the programs will be covered here.

A list of the programs and associated data files is given in Table E-1.

In a typical test the sequence of programs used would begin with the startup in which DWL 42 would be used to activate the loops. Also, RGM 43 may be required to reset the shutdown ECO which would restore power to the pumps.

The startup of the gas chromatograph requires an interrupt on level 8, bit 6. This is usually carried out by a hardware contact. The GC package queues a user program called DWL 44 which uses the peak areas to calculate the concentration of methanol.

The program DASS is used to initiate a material and energy balance calculation from direct measurements. It links to DATAC which links to BALNC. Results are used to ascertain if the column is at steady state.

Batch programs RGM 51 and RGM 54 are used to enter and write out data contained in files used by control programs. These programs DWL 52 and RGM 50 carry out the control calculation and inserts setpoints in DDC loop records as well as storing data from DDC for up to 124 minutes on a one minute sample time.

It is possible to commence another run almost immediately as the data must be copied to another file using RGM 46 for sorting. This frees the file used to store data accumulated during the run.

Finally, the data can be printed and punched and/or plotted using DWL 53.

TABLE E-1

ON-LINE PROGRAMS USED WITH DISTILLATION COLUMN

DASS, DATAC, BALNC - do material and energy balance from direct measurements from column.
- use disk files RGM 01 (1 sct) and PAC 01 (16 sct).

RGM 4A - writes labels to RGM 01 (for BALNC and DWL 53) (batch).

DWL 42 - turns column DDC loop on or off.

RGM 43 - turns shutdown ECO (re CRASH) on or off. DOUT (3,16,-)

DWL 44 - queued by GC Pak to calculate MEOH concentration and put in DDC loop 607. Uses CE Core words Z0017 and Z001B.

RGM 46 - copies data (collected by RGM 50) from file RGM 03 to RGM 04 for plotting, punching, etc. by DWL 53.

RGM 51 - enters control matrices into RGM 03 for RGM 50.

RGM 54 - writes out parameters for start run from RGM 03.

DWL 52 - starts and stops RGM 50 (timer 10) and sets initial conditions for explorational run. Writes in RGM 03 and to control word in CE Core Z91F9.

TABLE E-1 (Continued)

RGM 50 - interrupt coreload for control and data accumulation. (Runs 124 min.). Data goes to RGM 03 (17 sct).

DWL 53 - plots, punches (prints) data transferred to RGM 04 (16 sct).

SSDAT - batch job to calculate internal liquid and vapor flowrates and vapor compositions from feed, distribution, reflux flows and liquid compositions.

DWL 60 - batch coreload to do thesis plots.

APPENDIX F
CONTROLLER CONSTANTS

The controller constants used on the analog controllers on the experimental equipment are given in Table F-1.

Constants used in controllers for the overhead composition control tests in Chapter 2 are given in Table F-2.

TABLE F-1
ANALOG CONTROLLER CONSTANTS

CONTROLLER	PROPORTIONAL BAND (%)	RESET (MIN.) (INTEGRAL)
FEED FLOW	180	0.80
REFLUX FLOW	60	0.80
STEAM FLOW	210	0.15
CONDENSER LEVEL	35	8.
REBOILER LEVEL	125	10.
PRESSURE	200	1.
FEED TEMPERATURE	200	0.25
REFLUX TEMPERATURE	32	7.

SUPERVISORY DDC LOOPS

DDC LOOP	PROPORTIONAL CONSTANT (KP)	INTEGRAL CONSTANT (TI SEC.)
FEED FLOW (0601)	-0.9921	89.0
STEAM FLOW (0603)	-1.4921	120.5
DISTILLATE FLOW (0605)	-0.4921	100.0

TABLE F-2
OVERHEAD COMPOSITION CONTROLLER CONSTANTS FROM CHAPTER 2

CONSTANTS FOR FOXBORO CAPACITANCE DYNALOG

	PROPORTIONAL BAND (%)	RESET (MIN.) (INTEGRAL)
REFLUX FLOW USED AS MANIPULATED VARIABLE	200	7.0
DISTILLATE FLOW USED AS MANIPULATED VARIABLE	133	9.0

CONSTANTS FOR DDC CONTROL LOOPS

	PROPORTIONAL (KP)	INTEGRAL (TI SEC.)
REFLUX FLOW AS MANIPULATED VARIABLE	-0.60	128.
DISTILLATE FLOW AS MANIPULATED VARIABLE	0.50	256.
TUNED	0.75	600.